1.10 PROBLEMS

The following problems can be solved by writing commands in the Command Window, or by writing a program in a script file and then executing the file.

1. Calculate:

   (*a*) $\dfrac{22+5.1^2}{50-6.3^2}$ (*b*) $\dfrac{44}{7}+\dfrac{8^2}{5}-\dfrac{99}{3.9^2}$

2. Calculate:

   (*a*) $\dfrac{\sqrt{41^2-5.2^2}}{e^5-100.53}$ (*b*) $\sqrt[3]{132}+\dfrac{\ln(500)}{8}$

3. Calculate:

(*a*) $\dfrac{14.8^3 - 6.3^2}{(\sqrt{13}+5)^2}$ (*b*) $45\left(\dfrac{288}{9.3} - 4.6^2\right) - 1065e^{-1.5}$

4. Calculate:

(*a*) $\dfrac{24.5 + 64/3.5^2 + 8.3 \cdot 12.5^3}{\sqrt{76.4} - 28/15}$ (*b*) $(5.9^2 - 2.4^2)/3 + \left(\dfrac{\log_{10}12890}{e^{0.3}}\right)^2$

5. Calculate:

(*a*) $\cos\left(\dfrac{7\pi}{9}\right) + \tan\left(\dfrac{7}{15}\pi\right)\sin(15°)$ (*b*) $\sin^2 80° - \dfrac{(\cos 14° \sin 80°)^2}{\sqrt[3]{0.18}}$

6. Define the variable x as $x = 6.7$, then evaluate:

(*a*) $0.01x^5 - 1.4x^3 + 80x + 16.7$ (*b*) $\sqrt{x^3 + e^x - 51/x}$

7. Define the variable t as $t = 3.2$, then evaluate:

(*a*) $56t - 9.81\dfrac{t^2}{2}$ (*b*) $14e^{-0.1t}\sin(2\pi t)$

8. Define the variables x and y as $x = 5.1$ and $y = 4.2$, then evaluate:

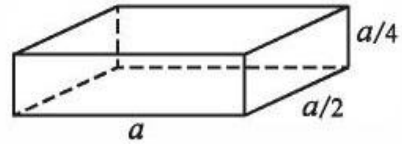
(*a*) $\dfrac{3}{4}xy - \dfrac{7x}{y^2} + \sqrt{xy}$ (*b*) $(xy)^2 - \dfrac{x+y}{(x-y)|^2} + \sqrt{\dfrac{x+y}{2x-y}}$

9. Define the variables a, b, c, and d as:

$a = 12$, $b = 5.6$, $c = \dfrac{3a}{b^2}$, and $d = \dfrac{(a-b)^c}{c}$, then evaluate:

(*a*) $\dfrac{a}{b} + \dfrac{d-c}{d+c} - (d-b)^2$ (*b*) $e^{\frac{d-c}{a-2b}} + \ln\left(\left|c - d + \dfrac{b}{a}\right|\right)$

10. A sphere has a radius of 24 cm. A rectangular prism has sides of a, $a/2$, and $a/4$.



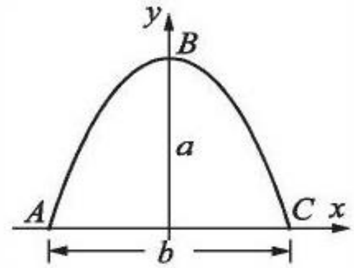
(*a*) Determine a of a prism that has the same volume as the sphere.

(*b*) Determine a of a prism that has the same surface area as the sphere.

11. The arc length of a segment of a parabola ABC of an ellipse with semi-minor axes a and b is given approximately by:

$$L_{ABC} = \frac{1}{2}\sqrt{b^2+16a^2} + \frac{b^2}{8a}\ln\left(\frac{4a+\sqrt{b^2+16a^2}}{b}\right).$$

(a) Determine L_{ABC} if $a = 11$ in. and $b = 9$ in.



12. Two trigonometric identities are given by:

(a) $\sin 5x = 5\sin x - 20\sin^3 x + 16\sin^5 x$ (b) $\sin^2 x\cos^2 x = \frac{1-\cos 4x}{8}$

For each part, verify that the identity is correct by calculating the values of the left and right sides of the equation, substituting $x = \frac{\pi}{12}$.

13. Two trigonometric identities are given by:

(a) $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$ (b) $\cos 4x = 8(\cos^4 x - \cos^2 x) + 1$

For each part, verify that the identity is correct by calculating the values of the left and right sides of the equation, substituting $x = 24°$.

14. Define two variables: *alpha* = π/6, and *beta* = 3π/8. Using these variables, show that the following trigonometric identity is correct by calculating the values of the left and right sides of the equation.

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

15. Given: $\int x\sin ax\,dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a}$. Use MATLAB to calculate the following definite integral: $\int_{\frac{\pi}{3}}^{\frac{3\pi}{2}} x\sin(0.6x)dx$.

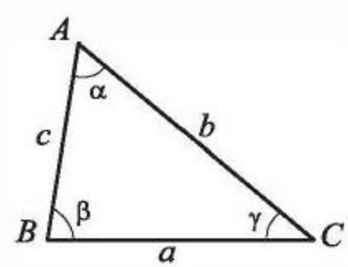
16. In the triangle shown $a = 5.3$ in., $\gamma = 42°$, and $b = 6$ in. Define a, γ, and b as variables, and then:

(a) Calculate the length b by using the Law of Cosines.

(Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$)

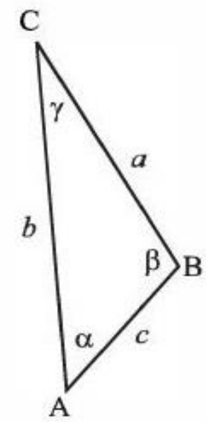
(b) Calculate the angles β and γ (in degrees) using the Law of Cosines.

(c) Check that the sum of the angles is 180°.

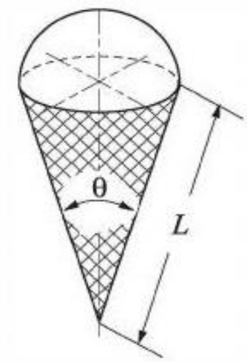


17. In the triangle shown $a = 5$ in., $b = 7$ in., and $\gamma = 25°$. Define a, b, and γ as variables, and then:
(*a*) Calculate the length of c by substituting the variables in the Law of Cosines.
(Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$)
(*b*) Calculate the angles α and β (in degrees) using the Law of Sines.
(*c*) Verify the Law of Tangents by substituting the results from part (*b*) into the right and left sides of the equation.

Law of Tangents: $\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\left[\frac{1}{2}(\alpha+\beta)\right]}$



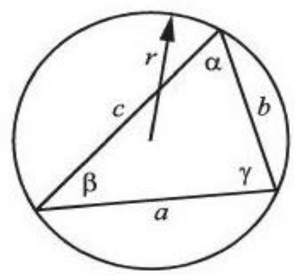
18. In the ice cream cone shown, $L = 4$ in. and $\theta = 35°$. The cone is filled with ice cream such that the portion above the cone is a hemisphere. Determine the volume of the ice cream.



19. For the triangle shown, $a = 48$ mm, $b = 34$ mm, and $\gamma = 83°$. Define a, b, and γ as variables, and then:
(*a*) Calculate c by substituting the variables in the Law of Cosines.
(Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$)
(*b*) Calculate the radius r of the circle circumscribing the triangle using the formula:

$$r = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

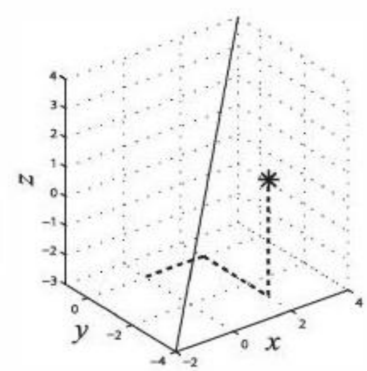
where $s = (a+b+c)/2$.



20. The parametric equations of a line in space are: $x = x_0 + at$, $y = y_0 + bt$, and $z = z_0 + ct$. The distance d from a point A (x_A, y_A, z_A) to the line can be calculated by:

$$d = d_{A0}\sin\left[\operatorname{acos}\left(\frac{(x_A-x_0)a + (y_A-y_0)b + (z_A-z_0)c}{d_{A0}\sqrt{a^2+b^2+c^2}}\right)\right]$$

where $d_{A0} = \sqrt{(x_A-x_0)^2 + (y_A-y_0)^2 + (z_A-z_0)^2}$.



Determine the distance of the point A (2, –3, 1) from the line $x = -4 + 0.6t$, $y = -2 + 0.5t$, and $z = -3 + 0.7t$. First define the variables x_0, y_0, z_0, a, b, and c, then use the variable (and the coordinates of point A) to calculate the variable d_{A0}, and finally calculate d.