## The Arithmetic Mean

The **arithmetic mean** is the average of the data set which is calculated by adding all the data values together and dividing it by the total number of data sets.

Or the **arithmetic mean** of a set of data is found by taking the sum of the data, and then dividing the sum by the total number of values in the set. A mean is commonly referred to as an average.

The *sample mean* of the values is $X_1, X_2, X_3, \ldots\ldots X_n$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} Xi = \frac{X1, X2, X3, \ldots .Xn}{n}$$

Example: find the sample mean for (78, 87, 68, 72, 91, 84)

$$\bar{x} = \frac{84+91+72+68+87+78}{6} = \frac{480}{80} = 80$$

Frequency data: suppose that the frequency of the class with midpoint $X_i$ is $f_i$ , for *i* = 1, 2, ..., m). Then

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{m} f_i x_i = \frac{f_1 x_1 + f_2 x_2 , + f_3 x_3, \ldots . f_m x_m}{n}$$

Where n = $\sum_{i=1}^{m} f_i$ = total number of observations.

Example:

Accidents data: find the sample mean.

| Number of accidents, xi | Frequency fi | fi xi |
|---|---|---|
| 0 | 55 | 0 |
| 1 | 14 | 14 |
| 2 | 5 | 10 |
| 3 | 2 | 6 |

| | | |
|---|---|---|
| 4 | 0 | 0 |
| 5 | 2 | 10 |
| 6 | 1 | 6 |
| 7 | 0 | 0 |
| 8 | 1 | 8 |
| TOTAL | 80 | 54 |

$\bar{X} = \frac{54}{80} = 0657$

Example 2:

| Classes | Fi | Xi | Fi* Xi |
|---|---|---|---|
| 31-40 | 1 | 35.5 | 35.5 |
| 41-50 | 2 | 45.5 | 91.0 |
| 51-60 | 5 | 55.5 | 277.5 |
| 61-70 | 15 | 65.5 | 282.5 |
| 71-80 | 25 | 75.5 | 1887.5 |
| 81-90 | 20 | 85.5 | 1710.0 |
| 91-100 | 12 | 95.5 | 1146.0 |
| | $\sum$= 80 | | $\sum$= 6130 |
| | | | |

$$\bar{\mathrm{X}} = \frac{\sum (\mathrm{f_i} x_i)}{\sum (\mathrm{f_i})} = \frac{6130}{80} = 76.62$$

## The harmonic mean

Is a very specific type of average. It’s generally used when dealing with averages of units, like speed or other rates and ratios.

The formula is:

$$H = \frac{n}{\frac{1}{x^1} + \frac{1}{x^2} + \cdots + \frac{1}{x^n}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{xi}}$$

*Examples*

What is the harmonic mean of 1, 5, 8, 10 ?

$$\boldsymbol{H = \frac{4}{\frac{1}{1}+\frac{1}{5}+\frac{1}{8}+\frac{1}{10}} = \frac{4}{1.425} = 2.80702}$$

Note: the harmonic mean is slightly less than the arithmetic mean.

## **The geometric mean:**

is a type of average , usually used for growth rates, like population growth or interest rates. While the arithmetic mean adds items, the geometric mean multiplies items. Also, you can only get the geometric mean for positive numbers.

$$\text{The geometric mean (GM)} = \sqrt[n]{X1 * X2 * X3 * Xn}$$

Example 1:

What is the geometric mean of 2,3, and 6?

First, multiply the numbers together and then take the cubed root (because there are three numbers) = (2*3*6)1/3 = 3.30

Example 2:

What is the geometric mean of 4,8.3,9 and 17?

First, multiply the numbers together and then take the 5th root (because there are 5 numbers) =

(4*8*3*9*17)(1/5) = 6.81

**Example 3:**

What is the geometric mean of 1/2, 1/4, 1/5, 9/72 and 7/4?

First, multiply the numbers together and then take the 5th root:

(1/2*1/4*1/5*9/72*7/4) (1/5) = 0.35.

**Example 4:**

The average person's monthly salary in a certain town jumped from $2,500 to $5,000 over the course of ten years. Using the geometric mean, what is the average yearly increase?

Solution:

Step 1: Find the geometric mean.

(2500*5000)^(1/2) = 3535.53390593.

Step 2: Divide by 10 (to get the average increase over ten years).

3535.53390593 / 10 = 353.53.

The average increase (according to the GM) is 353.53.

## The Quadratic Mean (R.M.S.):

The quadratic mean (also called the root mean square*) is a type of average.

The quadratic mean is also called the root mean square because it is the square root of the mean of the squares of the numbers in the set.

$$\textbf{R.M.S} = \sqrt{\frac{1}{n}\ (X1^2 + X2^2 \ldots\ldots\ldots. + X^2 n)}$$

Example:

Find the Quadratic Mean (R.M.S.) for the following data: (3,5,6,6,7,10,12)

$$R.M.S. = \sqrt{\frac{\sum(\bar{X})^2}{N}} = \sqrt{\frac{(3)^2 + (5)^2 + (6)^2 + (6)^2 + (7)^2 + (10)^2 + (12)^2}{7}}$$

$$= \sqrt{57} = 7.55$$

<u>The Quadratic Mean (R.M.S.) for grouped data:</u>

$$R.M.S. = \sqrt{\frac{\sum(Xi)^2 x\, fi}{\sum fi}}$$

**Example: Find the Quadratic Mean (R.M.S.) for the following data:**

| Classes | Frequency fi | Mid-Point Xi |
|---|---|---|
| 60 – 62 | 5 | 61 |
| 63 – 65 | 18 | 64 |
| 66 – 68 | 42 | 67 |
| 69 – 71 | 27 | 70 |
| 72 - 74 | 8 | 73 |

**The solution :**

| Classes | Frequency fi | Mid-Point Xi | $(Xi)^2$ | fi x $(Xi)^2$ |
|---|---|---|---|---|
| 60 – 62 | 5 | 61 | 3721 | 18605 |
| 63 – 65 | 18 | 64 | 4096 | 73728 |
| 66 – 68 | 42 | 67 | 4489 | 188538 |
| 69 – 71 | 27 | 70 | 4900 | 132300 |
| 72 - 74 | 8 | 73 | 5329 | 42632 |
| | $\sum$ 100 | | | $\sum$455803 |

$$R.M.S. = \sqrt{\frac{\sum(Xi)^2 x\, fi}{\sum fi}} = \sqrt{\frac{455803}{100}} = 67.51$$