Chapter (1) VECTORS

Introduction

Vectors are essential to physics and engineering. Many fundamental physical quantities are vectors, including displacement, velocity, force, and electric and magnetic vector fields. Scalar products of vectors define other fundamental scalar physical quantities, such as energy.
In introductory physics, vectors are Euclidean quantities that have geometric representations as arrows in one dimension (in a line), in two dimensions (in a plane), or in three dimensions (in space). They can be added, subtracted or multiplied. In this chapter, we explore elements of vector algebra for applications in mechanics and in electricity and magnetism. Vector operations also have numerous generalizations in other branches of physics.

1.1 Coordinate Systems

Many aspects of physics involve a description of a location in space. In Chapter 1, for example, we saw that the mathematical description of an object's motion requires a method for describing the object's position at various times. This description is accomplished with the use of coordinates, and in Chapter 1 we used the **Cartesian coordinate system**, in which horizontal and vertical axes intersect at a point defined as the origin (Fig. 1.1). Cartesian coordinates are also called ***rectangular coordinates***.

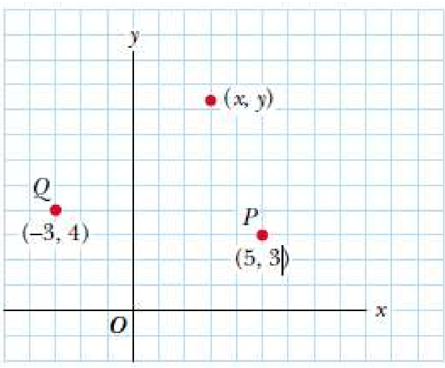


Figure 1.1 Designation of points in a Cartesian coordinate system. Every point is labeled with coordinates (x, y).

Sometimes it is more convenient to represent a point in a plane by its plane polar coordinates (**r**, **θ**), as shown in Figure1.2a. In this ***polar coordinate system***, r is the distance from the origin to the point having Cartesian coordinates (**x**, **y**), and **θ** is the angle between a line drawn from the origin to the point and a fixed axis. This fixed axis is usually the positive **x** axis, and **θ** is usually measured counter clockwise from it. From the right triangle in Figure 1.2b, we find that **sin θ = y/r** and that **cos θ = x/r**. Therefore, starting with the plane polar coordinates of any point, we can obtain the Cartesian coordinates by using the equations:

$$x = r\cos\theta \qquad (1-1)$$

$$y = r\sin\theta \qquad (1-2)$$

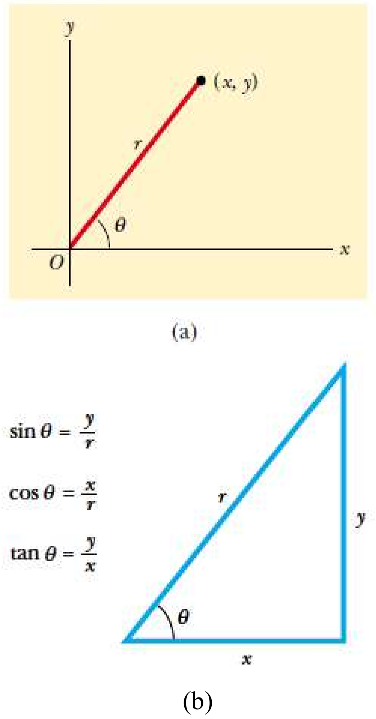


Figure **1.2** **(a)** The plane polar coordinates of a point are represented by the distance r and the angle θ, where θ is measured counterclockwise from the positive x axis. **(b)** The right triangle used to relate (x, y) to (r,θ).

Furthermore, the definitions of trigonometry tell us that

$$\tan\theta = \frac{y}{x} \qquad (1-3)$$

$$r = \sqrt{x^2 + y^2} \qquad (1-4)$$

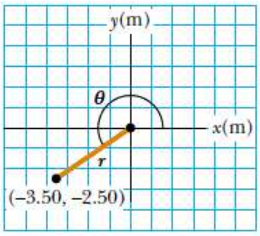
Equation 1.4 is the familiar Pythagorean theorem.

These four expressions relating the coordinates (x, y) to the coordinates (r, θ) apply only when θ is defined as shown in Figure 1.2a—in other words, when positive θ is an angle measured counterclockwise from the positive x axis. (Some scientific calculators perform conversions between Cartesian and polar coordinates based on these standard conventions.) If the reference axis for the polar angle θ is chosen to be one other than the positive x axis or if the sense of increasing θ is chosen differently, then the expressions relating the two sets of coordinates will change.

Example 1.1 Polar Coordinates

The Cartesian coordinates of a point in the xy plane are $(x, y) = (-3.50, -2.50)$ m, as shown in Figure 3.3. Find the polar coordinates of this point.

Solution For the examples in this and the next two chapters we will illustrate the use of the General Problem-Solving Strategy outlined at the end of Chapter 2. In subsequent chapters, we will make fewer explicit references to this strategy, as you will have become familiar with it and should be applying it on your own. The drawing in Figure 3.3 helps us to *conceptualize* the problem. We can *categorize* this as a plug-in problem. From Equation 3.4,



Active Figure 3.3 (Example 3.1) Finding polar coordinates when Cartesian coordinates are given.

At the Active Figures link at http://www.pse6.com, you can move the point in the xy plane and see how its Cartesian and polar coordinates change.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2} = 4.30 \text{ m}$$

and from Equation 3.3,

$$\tan\theta = \frac{y}{x} = \frac{-2.50 \text{ m}}{-3.50 \text{ m}} = 0.714$$

$$\theta = 216°$$

Note that you must use the signs of x and y to find that the point lies in the third quadrant of the coordinate system. That is, $\theta = 216°$ and not $35.5°$.

1.2 Vector and Scalar Quantities

As noted in our studies some physical quantities are scalar quantities whereas others are vector quantities. When you want to know the temperature outside so that you will know how to dress, the only information you need is a number and the unit “degrees C” or “degrees F.” Temperature is therefore an example of a scalar quantity:

A scalar quantity is completely specified by a single value with an appropriate unit and has no direction.

Other examples of scalar quantities are volume, mass, speed, and time intervals. The rules of ordinary arithmetic are used to manipulate scalar quantities.

If you are preparing to pilot a small plane and need to know the wind velocity, you must know both the speed of the wind and its direction. Because direction is important for its complete specification, velocity is a vector quantity:

A **vector quantity** is completely specified by a number and appropriate units plus a direction.

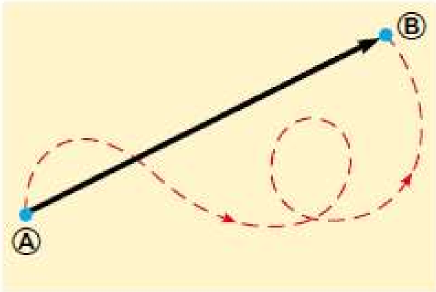


Figure 1.3 As a particle moves from (A) to (B) along an arbitrary path represented by the broken line, its displacement is a vector quantity shown by the arrow drawn from (A) to (B).

Quick Quiz 1.1 Which of the following are vector quantities and which are scalar quantities? (a) your age (b) acceleration (c) velocity (d) speed (e) mass

1.3 Some Properties of Vectors

Equality of Two Vectors

For many purposes, two vectors **A** and **B** may be defined to be equal if they have the same magnitude and point in the same direction. That is, **A** = **B** only if A = B and if **A** and **B** point in the same direction along parallel lines. For example, all the vectors in Figure 1.4 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.

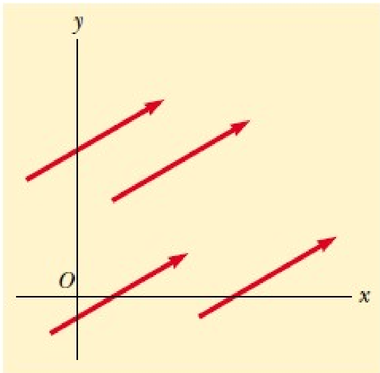


Figure 1.4 These four vectors are equal because they have equal lengths and point in the same direction.

Adding Vectors

The rules for adding vectors are conveniently described by graphical methods. To add vector B to vector A, first draw vector A on graph paper, with its magnitude represented by a convenient length scale, and then draw vector B to the same scale with its tail starting from the tip of A, as shown in Figure 1.5. **The resultant vector R = A + B** is the vector drawn from the tail of A to the tip of B.

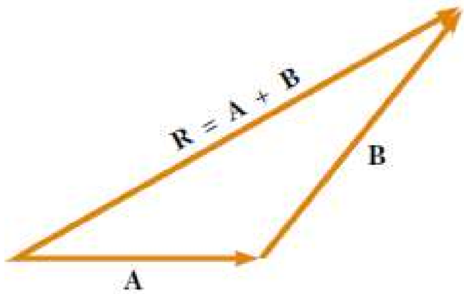


Figure 1.5 When vector **B** is added to vector **A**, the resultant **R** is the vector that runs from the tail of **A** to the tip of **B**.

For example, if you walked 3.0 m toward the east and then 4.0 m toward the north, as shown in Figure 1.6, you would find yourself 5.0 m from where you started, measured at an angle of 53° north of east. Your total displacement is the vector sum of the individual displacements.

A geometric construction can also be used to add more than two vectors. This is shown in Figure 1.7 for the case of four vectors. The resultant vector **R = A + B + C + D** is the vector that completes the polygon. In other words, **R is the vector drawn from the tail of the first vector to the tip of the last vector.**

When two vectors are added, the sum is independent of the order of the addition.

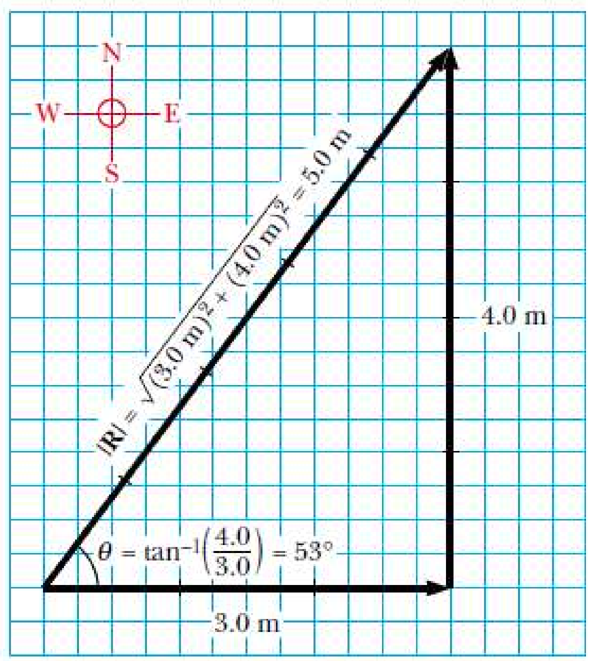


Figure 1.6 Vector addition. Walking first 3.0 m due east and then 4.0m due north leaves you 5.0 m from your starting point.

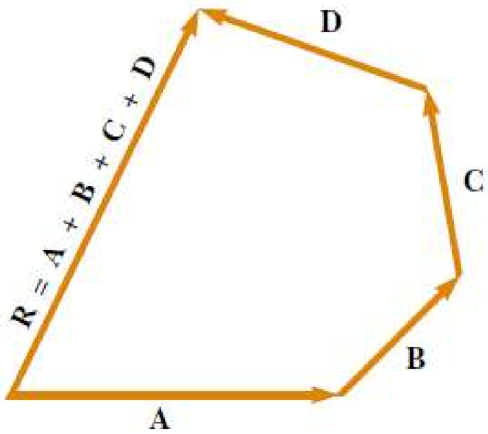


Figure 1.7 Geometric construction for summing four vectors. The resultant vector **R** is by definition the one that completes the polygon.

This can be seen from the geometric construction in Figure 1.8 and is known as the commutative law of addition:

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \qquad (1-5)$$

When three or more vectors are added, their sum is independent of the way in which the individual vectors are grouped together. A geometric proof of this rule for three vectors is given in Figure 1.9. This is called **the associative law of addition**:

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C} \qquad (1-6)$$

In summary, a vector quantity has both magnitude and direction and also obeys the laws of vector addition as described in Figures 1.5 to 1.9. When two or more vectors are added together, all of them must have the same units and all of them must be the same type of quantity. It would be meaningless to add a velocity vector (for example, 60 km/h to the east) to a displacement vector (for example, 200 km to the north) because they represent different physical quantities. The same rule also applies to scalars. For example, it would be meaningless to add time intervals to temperatures.

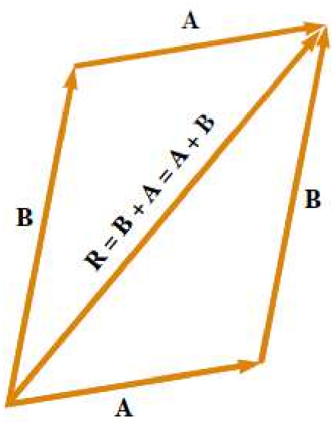


Figure 1.8 This construction shows that **A+ B = B + A** in other words, that vector addition is commutative.