Negative of a Vector

The negative of the vector A is defined as the vector that when added to A gives zero for the vector sum. That is, **A + (-A) = 0**. The vectors **A** and **-A** have the same magnitude but point in opposite directions.

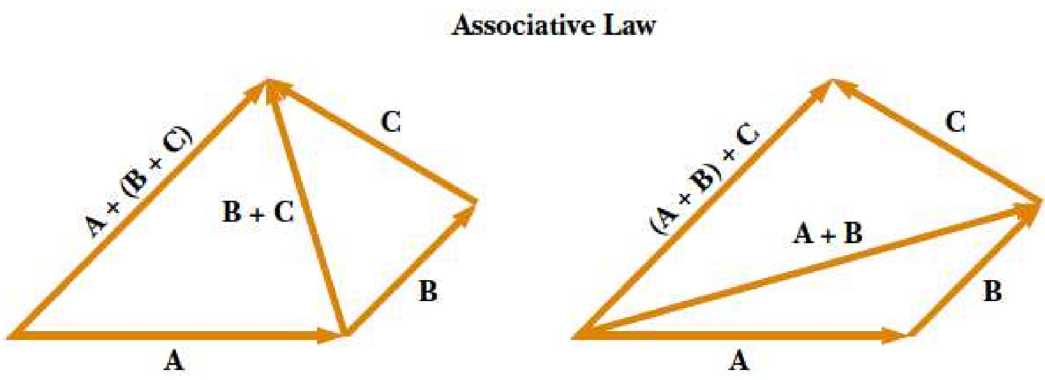


Figure 1.9 Geometric constructions for verifying the associative law of addition.

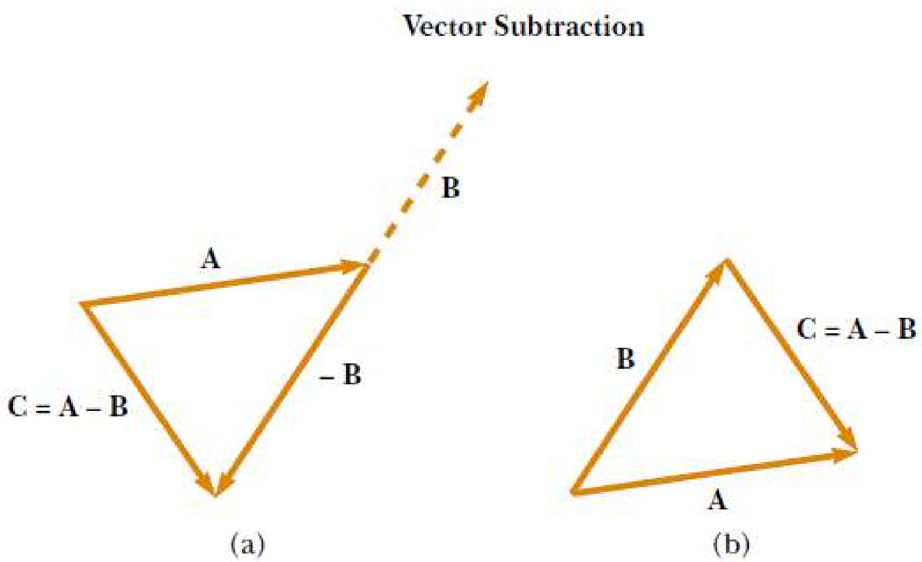


Figure 1.10 (a) This construction shows how to subtract vector **B** from vector **A**. The vector **-B** is equal in magnitude to vector B and points in the opposite direction. To subtract **B** from **A**, apply the rule of vector addition to the combination of A and **-B**: Draw **A** along some convenient axis, place the tail of **-B** at the tip of **A**, and C is the difference **A - B**. (b) A second way of looking at vector subtraction. The difference vector **C = A - B** is the vector that we must add to **B** to obtain **A**.

Subtracting Vectors

The operation of vector subtraction makes use of the definition of the negative of a vector. We define the operation **A - B** as vector **-B** added to vector **A**:

$$\mathbf{A} - \mathbf{B} = \mathbf{A}(-\mathbf{B}) \qquad (1-7)$$

The geometric construction for subtracting two vectors in this way is illustrated in Figure 1.10a. Another way of looking at vector subtraction is to note that the difference **A - B** between two vectors **A** and **B** is what you have to add to the second vector to obtain the first. In this case, the vector **A - B** points from the tip of the second vector to the tip of the first, as Figure 1.10b shows.

Quick Quiz 1.2 The magnitudes of two vectors A and B are A = 12 units and B = 8 units. Which of the following pairs of numbers represents the largest and smallest possible values for the magnitude of the resultant vector R = A + B? (a) 14.4 units, 4 units (b) 12 units, 8 units (c) 20 units, 4 units (d) none of these answers.

Quick Quiz 1.3 If vector B is added to vector A, under what condition does the resultant vector A + B have magnitude A + B? (a) A and B are parallel and in the same direction. (b) A and B are parallel and in opposite directions. (c) A and B are perpendicular.

Quick Quiz 1.4 If vector **B** is added to vector **A**, which *two* of the following choices must be true in order for the resultant vector to be equal to zero? (a) **A** and **B** are parallel and in the same direction. (b) **A** and **B** are parallel and in opposite directions. (c) **A** and **B** have the same magnitude. (d) **A** and **B** are perpendicular.

Example 1.2 **A Vacation Trip**

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north, as shown in Figure 1.11a. Find the magnitude and direction of the car's resultant displacement.

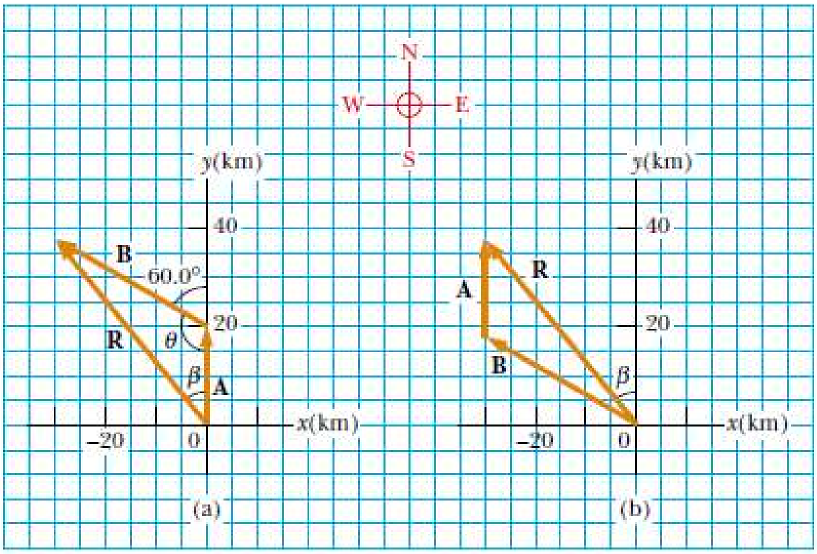


Figure 1.9 (Example 1.2) (a) Graphical method for finding the resultant displacement vector **R** = **A** + **B**.
(b) Adding the vectors in reverse order (**B** + **A**) gives the same result for **R**.

Solution

The vectors A and B drawn in Figure 1.11a help us to *conceptualize* the problem. We can *categorize* this as a relatively simple analysis problem in vector addition. The displacement R is the resultant when the two individual displacements A and B are added. We can further categorize this as a problem about the analysis of triangles, so we appeal to our expertise in geometry and trigonometry.

In this example, we show two ways to *analyze* the problem of finding the resultant of two vectors. The first way is to solve the problem geometrically, using graph paper and a protractor to measure the magnitude of R and its direction in Figure 1.11a. (In fact, even when you know you are going to be carrying out a calculation, you should sketch the vectors to check your results.) With an ordinary ruler and protractor, a large diagram typically gives answers to two-digit but not to three-digit precision.

The second way to solve the problem is to analyze it algebraically. The magnitude of R can be obtained from the law of cosines as applied to the triangle. With $\theta = 180° - 60° = 120°$ and $R^2 = A^2 + B^2 - 2AB\cos\theta$, we find that

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$
$$= \sqrt{(20.0\text{ km})^2 + (35.0\text{ km})^2 - 2(20.0\text{ km})(35.0\text{ km})\cos 120°}$$
$$= 48.2\text{ km}$$

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta = \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\beta = 39.0°$$

Multiplying a Vector by a Scalar

If vector A is multiplied by a positive scalar quantity *m*, then the product *m*A is a vector that has the same direction as A and magnitude *mA*. If vector A is multiplied by a negative scalar quantity -*m*, then the product -*m*A is directed opposite A. For example, the vector 5A is five times as long as A and points in the same direction as A; the vector -1/3A is one-third the length of A and points in the direction opposite A.

1.4 Components of a Vector and Unit Vectors

The graphical method of adding vectors is not recommended whenever high accuracy is required or in three-dimensional problems. In this section, we describe a method of adding vectors that makes use of the projections of vectors along coordinate axes. These projections are called the **components** of the vector. Any vector can be completely described by its components.

Consider a vector **A** lying in the ***xy*** plane and making an arbitrary angle θ with the positive ***x*** axis, as shown in Figure 1.12a. This vector can be expressed as the sum of two other vectors **A*x*** and **A*y***.

From Figure 1.12b, we see that the three vectors form a right triangle and that **A** = **A*x*** + **A*y***. We shall often refer to the “components of a vector A,” written A*x* and A*y* (without the boldface notation). The component A*x* represents the projection of **A** along the *x* axis, and the component A*y* represents the projection of **A** along the y axis. These components can be positive or negative. The component A*x* is positive if A*x* points in the positive x direction and is negative if A*x* points in the negative x direction. The same is true for the component Ay.

From Figure 1.12 and the definition of sine and cosine, we see that cos θ = A*x*/A and that sin θ " Ay/A. Hence, the components of **A** are

$$A_x = A\cos\theta \qquad (1-8)$$

$$A_y = A \sin \theta \qquad (1-9)$$

These components form two sides of a right triangle with a hypotenuse of length *A*. Thus, it follows that the magnitude and direction of A are related to its components through the expressions:

$$A = \sqrt{{A_x}^2 + {A_y}^2} \qquad (1-10)$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \qquad (1-11)$$

Note that **the signs of the components A*x* and A*y* depend on the angle θ**. For example, if **θ** = 120°, then A*x* is negative and A*y* is positive. If **θ** = 225°, then both A*x* and A*y* are negative. Figure 1.13 summarizes the signs of the components when A lies in the various quadrants.
When solving problems, you can specify a vector **A** either with its components A*x* and A*y* or with its magnitude and direction A and **θ**.

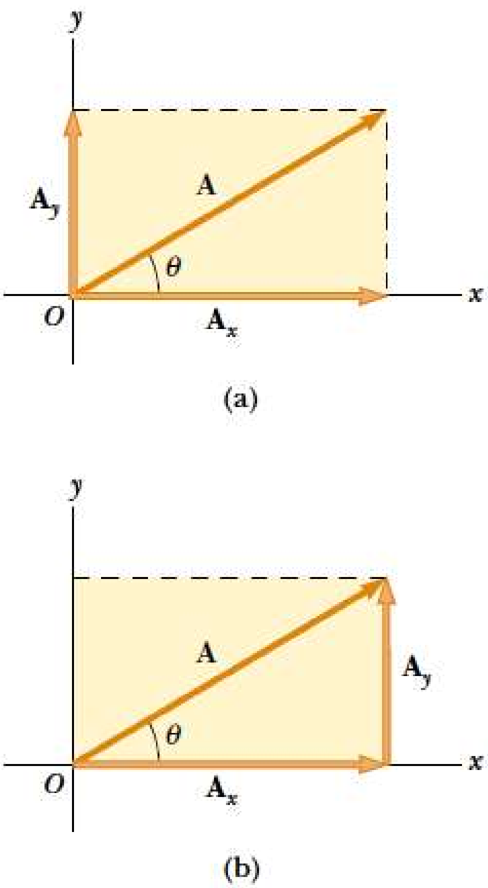


Figure 1.12 (a) A vector **A** lying in the *xy* plane can be represented by its component vectors **A***x* and **A***y* . (b) The *y* component vector **A***y* can be moved to the right so that it adds to **A***x* . The vector sum of the component vectors is **A**. These three vectors form a right triangle.

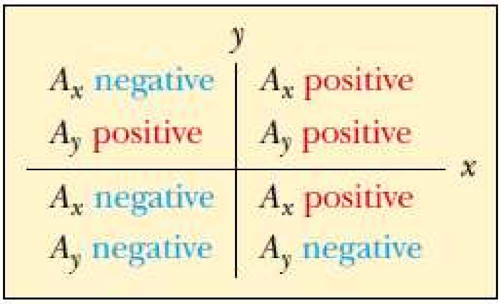


Figure 1.13 The signs of the components of a vector **A** depend on the quadrant in which the vector is located.

Unit Vectors

1

Vector quantities often are expressed in terms of unit vectors. **A unit vector is a dimensionless vector having a magnitude of exactly 1.** Unit vectors are used to specify a given direction and have no other physical significance. They are used solely as a convenience in describing a direction in space. We shall use the symbols $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ to represent unit vectors pointing in the positive x, y, and z directions, respectively. (The "hats" on the symbols are a standard notation for unit vectors.) The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ form a set of mutually perpendicular vectors in a right-handed coordinate system, as shown in Figure **1.15a** The magnitude of each unit vector equals 1; that is, $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$.

Consider a vector $\mathbf{A}$ lying in the xy plane, as shown in Figure **1.15b** . The product of the component A_x and the unit vector $\hat{\mathbf{i}}$ is the vector $A_x\hat{\mathbf{i}}$, which lies on the x axis and has magnitude $|A_x|$. (The vector $A_x\hat{\mathbf{i}}$ is an alternative representation of vector $\mathbf{A}_x$.) Likewise, $A_y\hat{\mathbf{j}}$ is a vector of magnitude $|A_y|$ lying on the y axis. (Again, vector $A_y\hat{\mathbf{j}}$ is an alternative representation of vector $\mathbf{A}_y$.) Thus, the unit–vector notation for the vector $\mathbf{A}$ is

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \qquad \textbf{(1-12)}$$

For example, consider a point lying in the xy plane and having Cartesian coordinates (x, y), as in Figure 3.17. The point can be specified by the **position vector r**, which in unit–vector form is given by

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} \qquad \textbf{(1.13)}$$

This notation tells us that the components of $\mathbf{r}$ are the lengths x and y.

Now let us see how to use components to add vectors when the graphical method is not sufficiently accurate. Suppose we wish to add vector $\mathbf{B}$ to vector $\mathbf{A}$ in Equation 3.12, where vector $\mathbf{B}$ has components B_x and B_y. All we do is add the x and y components separately. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is therefore

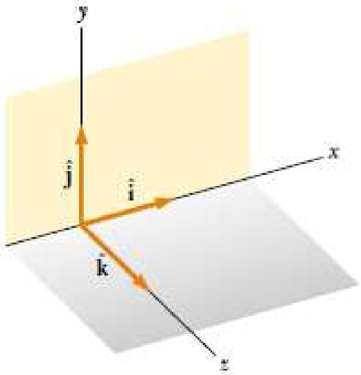
$$\mathbf{R} = (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}) + (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}})$$

or

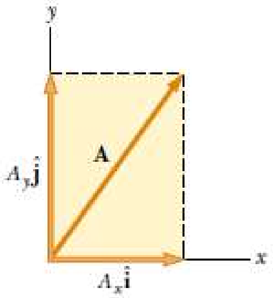
$$\mathbf{R} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} \qquad \textbf{(1-14)}$$

Because $\mathbf{R} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$, we see that the components of the resultant vector are

$$R_x = A_x + B_x \qquad \textbf{(1-15)}$$
$$R_y = A_y + B_y$$



(a)



(b)

Active Figure 1.15 (a) The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are directed along the x, y, and z axes, respectively. (b) Vector $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$ lying in the xy plane has components A_x and A_y.