Chapter (3) Motion in One Dimension

1-Average velocity

Let us think about a material object (a "particle") which is constrained to move along a given straight line.

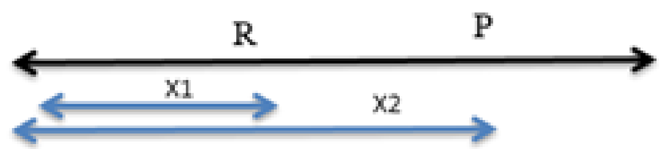
The position of the particle at any instant can be specified by a number X which gives the distance from the origin to the particle. The numerical value of X clearly depends on the unit of length we are using (e.g. feet, meters, or miles).

Unless the particle is at rest, X will vary with time. When the particle at point P at position X_1 and time t_1 if the particle move to point R at position X_2 and time t_2 .The value of X at time t is denoted by X (t).

The average velocity of a particle during the time interval from t_1 to t_2 is defined the ratio of the displacement to the time interval and given by equation (1-1).

$$v_{avg} = \frac{X_2 - X_1}{t_2 - t_1} = \frac{\Delta X}{\Delta t} \quad ......(1\text{-}1)$$

i.e. the change in position divided by the change in time. If we draw a graph of x versus t (for example, Fig.1.1).



Can be write Equation (1-1) as:

$$X_2 = X_1 + v_{avg}(t_2 - t_1) \ldots\ldots (1-2)$$

Where **the average Speed** is, define as the length of path by the time. **The average velocity** is vector quantity but the average soeed is scalar quantity.

2- Instantaneous velocity

The velocity of particle at some one instant of time or some one point its path is called **instantaneous velocity**. Let the average velocity be computed over these shorter and shorter displacement and time interval. In the notation of calculus the limiting value of $\frac{\Delta x}{\Delta t}$ at Δt Approches Zero. This limiting value, which may be thought of as the average velocity over an infinitesimal time interval which includes the time t, is called the "the instantaneous velocity at time t" or, more briefly, "the velocity at time t". We write.

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \ldots\ldots\ldots..(1-3)$$

This equation is familiar to anyone who has studied differential calculus; the right side is called (the derivative of x with respect to t) and frequently denoted by dx/dt. Thus the velocity is V= dx/dt. If x (t) is given in the form of an explicit formula, we can calculate v (t) either directly from equation (1-3).

3- Acceleration

Acceleration is defined as the velocity changes with time. The average acceleration during the interval from t_1 to t_2 is defined as the ratio of the change in velocity to elapsed time. We shall represent average acceleration by the symbol $(\vec{a})$ or a_{ave} is defined as:

$a_{ave} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{dV}{dt}$ ……… (1-4) where the acceleration is vector quantity.

4- Rectilinear motion with constant acceleration

We have a= dv/dt

$\int_{v_0}^{v} dv = a \int_{t_0}^{t} dt$

$V = V_0 + a\,(t - t_0)$ if $t_0 = 0$

$V = V_0 + at$ ………… (1)

Also we have V= dv/dt but in equ. (1) We get

$\frac{dX}{dt} = V_0 + at$

$dx = (V_0 + at)\,dt$

$\int_{x_0}^{x} dx = V_0 \int_{t_0}^{t} dt + a \int_{t_o}^{t} dt$

Where $X_0 = 0$, $t_0 = 0$ we get

$X = V_0 t + 1/2\ at^2$ …………… (2) We get from this equ.

$X = 1/2\ t\ (V_0 + V)$ ……………. (3) We have

$$a = \frac{dv}{dt} = \frac{dv}{dt} \cdot \frac{dx}{dx} = \frac{dv}{dx} \cdot \frac{dx}{dt} \quad \frac{dv}{dx} \cdot V$$

$V\ dv = a\ dx$

$$\int_{v_0}^{v} v dv = a \int_{x_0}^{x} dx$$

$$\frac{1}{2} V^2 - \frac{1}{2} V_0^2 = a(x - x_o)$$

$$V^2 = V_0^2 + 2a\ (x - x_0)$$

Where $X_0 = 0$ We get

$$V^2 = V_0^2 + 2ax \quad \dots\dots (4)$$

4- Freely Falling Bodies

It is an experimental fact that in the vicinity of a given point on the earth's surface, and in the absence of air resistance, all objects fall with the same constant acceleration. The magnitude of the acceleration is called g and is approximately equal to **9.8 meters/sec^2 or 980 cm/sec^2**, and the direction of the acceleration is down, i.e. toward the center of the earth.

The acceleration due to gravity is denoted by the letter **g**. One should orient the axes in the way which is mathematically most convenient. We let the positive y-axis point vertically up (i.e. away from the center of the earth). We change symbol (X…..Y, a.….g) and $X \neq 0$ the last equation will be

$V = V_0 - gt$

$y = y_0 + V_0 t - 1/2\ g\ t^2$

$y = y_0 + 1/2\ t\ (V - V_0)$

$V_2 = V_0^2 - 2g\ (y - y_0)$

Example (3-1)

A body moving on the X axis, distance (x) is given by the equation

$X=50\ t + 10\ t^2$

Find (a) – The average velocity of the body in the interval from t= 0 to

t = 3

(b) The instantaneous velocity at t =3 sec

(c) The instantaneous acceleration at t =3 sec

Solution: (a)

$$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

X_1 at t = 0

$X_1 = 50\ t_0 + 10\ t_0 = 0$ at t =3 sec

$X_2 = 50 * 3 + 10 * 3^2 = 240$ m

80m/sec $= \frac{240}{3}$ $V_{avg} = \frac{240-0}{3-0}$

(b)

$$V = \frac{dX}{dt} = 50 + 20t$$

at t = 3sec

V= 50 + 20*3 = 110 m/sec

(c) $a = \frac{dv}{dt} = 20$

a = 20 m/sec

Example (3-2)

A body moving on the X axis, its distance is given by the equation

$X = 2t^3 + 5t^2 + 5$

Find (a) the velocity and acceleration of body at any time

(b) The position, velocity and acceleration after 2 and 3 seconds

(c) The velocity and acceleration between 2 and 3 second.

Solution: (a)

$X = 2t^3 + 5t^2 + 5$

$$\frac{dx}{dt} = \frac{d}{dt}(2t^3 + 5t^2 + 5)$$

$V = 6t^2 + 10t$ m/sec

$$\frac{dv}{dt} = \frac{d}{dt}(6t^2 + 10t)$$

a = 12t + 10 m/sec

(b) at 2 sec

$$x = 2(2)^3 + 5(2)^2 + 5 = 41\, m$$

$$V = 6(2)^2 + 10(2) = 44\frac{m}{sec}$$

$a = 12(2) + 10 = 34$ m/sec^2

and the same solution we get the position velocity and acceleration after 3 second.

X = 104 m

V = 84 m/sec

a = 46 m/sec^2

(c) we have t= 3 sec , t = 2 sec

$$\Delta t = 3 - 2 = 1sec$$

From (B) we get $\Delta x\ 104 - 41 = 63m$

$$\Delta V = 84 - 44 = 40 \frac{m}{sec}$$

$$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{63}{1} = 63 \frac{m}{sec}$$

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{40}{1} = 40 \frac{m}{sec^2}$$

Example (3-3)

A ball is thrown up word from hand baby with a velocity of 14.7 m/sec. The height from earth at time learning hand its 1.5m. Find the maximum height reached ball and the time at which it is reached.

Solution:

$V2 = V_0^2 - 2g (y - y_0)$

$0 = (14.7)^2 - 2 * 9.8 (y - 1.5)$

$y = 12.5$ m

$V = V_0 - gt$

$0 = 14.7 - 9.8t$

$t = 14.7 / 9.8 = 1.5$ sec

Example (4)

A body moving on the X axis its acceleration (a) is given by the equation

a = 4x + 2

Where X is meters and V0 =10 m/sec where X_0= 0

Find the velocity at anther distance.

Solution:

We have a = 4x +2

$$\frac{vdv}{dx} = 4x + 2$$

$$\int_{v_0}^{v} vdv = \int_{x_0}^{x} 4xdx + \int_{x_0}^{x} 2dx$$

$$\int_{10}^{v} vdv = \int_{0}^{x} 4xdx + \int_{0}^{x} 2dx$$

$$\frac{v^2}{2} - \frac{10^2}{2} = 2x^2 + 2x$$

$$V = \sqrt{4x^2 + 4x + 100}$$