Example:- Find the solution $y' = -y$, $y(0) = 1$

Solution:-

$y' = -y = f(t,y)$

$\therefore f(t,y) = -y$

$f(s, \phi_{j-1}(s)) = -\phi_{j-1}(s)$.

$\phi_j(t) = y_0 + \int_{t_0}^{t} f(s, \phi_{j-1}(s)).ds$, $j = 1, 2, \ldots$

where

$j = 1$

$\phi_1(t) = y_0 + \int_0^t f(s, \underbrace{\phi_0(s)}_{=y_0}).ds$

$\phi_1(t) = 1 + \int_0^t (-\phi_0(s)).ds$

$= 1 - \int_0^t ds$

$= 1 - [s]_0^t = 1 - t$.

if $j = 2$

$\phi_2(t) = 1 + \int_0^t f(s, \phi_1(s)).ds$

$= 1 + \int_0^t (-\phi_1(s)).ds$

$= 1 - \int_0^t (1 - s).ds$

$= 1 - [s - s^2/2]_0^t$

$= 1 - (t - t^2/2)$.

if $j = 3$

$\phi_3(t) = 1 + \int_0^t f(s, \phi_2(s)).ds$

$= 1 + \int_0^t (-\phi_2(s)).ds$

$= 1 - \int_0^t (1 - s + s^2/2).ds$

$= 1 - [s - s^2/2 + s^3/6]_0^t$

$= 1 - (t - t^2/2 + t^3/6)$

$= 1 - \left(\frac{t}{1!} - \frac{t^2}{2!} + \frac{t^3}{3!}\right)$

$$\phi_4(t) = 1 + \int_0^t f(s, \phi_3(s))\, ds$$
$$= 1 - t + \frac{t^2}{2} - \frac{t^3}{3!} + \frac{t^4}{4!}.$$

$$\phi_k(t) = 1 - \frac{t}{1!} + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots \frac{(-1)^k t^k}{k!}$$

$$\lim_{k \to \infty} \phi_k(t) = \sum_{k=0}^{\infty} \frac{(-1)^k t^k}{k!} = e^{-t}$$

الحل الدقيق كالاتي

$$y' = -y \Rightarrow \frac{dy}{dt} = -y \Rightarrow \frac{dy}{y} = -dt.$$
$$\ln y = -t + c$$
$$y = e^{c_1 - t}$$
$$1 = e^{c_1} \Rightarrow y = e^{-t}.$$

Example:-

$$y' = 2x - y/x, \quad y(1) = 2.$$

Solution:- $x_0 = 1, \ \phi_0 = 2.$

$$\phi_j(x) = y_0 + \int_{x_0}^{x} f(s, \phi_{j-1}(s))\, ds$$

$$f(x, y) = y' = 2x - y/x$$

$x \to s$, $y \to \phi_{j-1}(s)$.

$$f(s, \phi_{j-1}(s)) = 2s - \frac{\phi_{j-1}(s)}{s}$$

$$\phi_j(x) = 2 + \int_1^x \left[2s - \frac{\phi_{j-1}(s)}{s}\right] ds.$$

$$\text{if } j = 1, \quad \phi_1(x) = 2 + \int_1^x \left(2s - \frac{\phi_0(s)}{s}\right) ds$$
$$= 2 + \int_1^x (2s - 2/s)\, ds$$
$$= 2 + \left[\frac{2}{2}s^2 - 2\ln s\right]_1^x$$
$$= 2 + \left[(x^2 - \ln x^2) - (1 - 0)\right]$$

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$= 2 + x^2 - \ln x^2 - 1$

$\phi_1(x) = 1 + x^2 - \ln x^2$

$\phi_2(x) = 2 + \int_1^x \left(2s - \frac{\phi_1(s)}{s}\right) . ds$

$= 2 + \int_1^x \left[2s - \frac{1+s^2 - \ln s^2}{s}\right] . ds$

$= 2 + \int_1^x (2s) . ds - \int_1^x \left(\frac{1}{s} - s - \ln \frac{s^2}{s}\right) . ds$

$2 + [s^2]_1^x + [-\ln s + \frac{s^2}{2} - (\ln s)^2]_1^x$

Example:- $y' = 2, \quad y(0) = 0$ ch.

يمكن أن تكون تدريبات منزلية لبيكارد.

Example:- Find the integral equation which evalent to the I.V.P. $y'' + w^2 y = f(t, y)$.

Solution:-

$y(0) = y_0, \quad y'(0) = z_0.$

$y'' + w^2 y = 0$

$m^2 + w^2 = 0 \Rightarrow m_1 = wi, \quad m_2 = -wi$

$y = A\cos wt + B\sin wt.$

$\phi_1(t) = \cos wt, \quad \phi_2(t) = \sin wt.$

$$W(\phi_1, \phi_2)(t) = \begin{vmatrix} \cos wt & \sin wt \\ -w\sin wt & \cos wt \end{vmatrix}.$$

$= w\cos^2 wt + w\sin^2 wt = w \neq 0.$

$$u_1(t) = -\int_{t_0}^{t} \frac{\phi_2(s) . f(s, y) . ds}{a_0(s)\, W(\phi_1, \phi_2)(s)}$$

$$= -\frac{\int_0^t \sin ws \cdot f(s,y) \cdot ds}{1 \cdot w} = \frac{-1}{w}\int_0^t f(s,y)\, \sin ws \cdot ds.$$

$$u_2(t) = \int_{t_0}^{t} \frac{\phi_1(s) \cdot f(s,y) \cdot ds}{a_0(s) \cdot W(\phi_1, \phi_2)(s)}$$

$$= \frac{\int_0^t \cos ws \cdot f(s,y) \cdot ds}{1 \cdot w} = \frac{1}{w}\int_0^t f(s,y) \cdot \cos ws \cdot ds.$$

$$y_p = u_1\phi_1(t) + u_2\phi_2(t) = \frac{-1}{w}\int_0^t f(s,y) \cdot \sin ws \cdot \cos wt \cdot ds$$

$$+ \frac{1}{w}\int_0^t f(s,y) \cdot \cos ws \cdot \sin wt \cdot ds.$$

$$y_p = \frac{1}{w}\int_0^t f(s,y) \cdot \sin w(t-s) \cdot ds$$

$$y = A\cos wt + B\sin wt + \frac{1}{w}\int_0^t f(s,y) \cdot \sin w(t-s) \cdot ds.$$

Ex

$$\frac{dy}{dx} = 2x - \frac{y}{x}$$

$$\frac{dy}{dx} + \frac{y}{x} = 2x$$

$$I.F = e^{\int \frac{1}{x} \cdot dx} = e^{\ln x} = x$$

$$x \cdot \frac{dy}{dx} + \frac{y}{\cancel{x}} \cdot \cancel{x} = 2x^2$$

$$\frac{d}{dx}(x \cdot y) = 2x^2$$

$$\int d(x \cdot y) = \int (2x^2) \cdot dx$$

$$x \cdot y = \frac{2}{3}x^3$$

$$y = \frac{2}{3}x^2$$