

Example (CDF)

if X is a random variable with values $-1, 1, 2$, and 3 , with probabilities $\frac{1}{4}, \frac{1}{8}, \frac{1}{8}$, and $\frac{1}{2}$, respectively. We then

have

$$F_X(x) = \begin{cases} 0 = P(X < -1) & \text{if } x < -1 \\ \frac{1}{4} = P(-1 \leq X < 1) & \text{if } -1 \leq x < 1 \\ \frac{3}{8} = P(1 \leq X < 2) & \text{if } 1 \leq x < 2 \\ \frac{1}{2} = P(2 \leq X < 3) & \text{if } 2 \leq x < 3 \\ 1 = P(X \geq 3) & \text{if } x \geq 3 \end{cases}$$

نوٹ کریں :-

$$P(X < -1) = P(\emptyset) = 0$$

$$P(-1 \leq X < 1) = P(X \in \{-1\}) = \frac{1}{4}$$

$$\begin{aligned} (-1 \leq X < 1) &= \{\omega : X(\omega) < 1, X(\omega) \geq -1\} \\ &= \{\omega : X(\omega) = -1\} = \{X = -1\} \end{aligned}$$

$$\therefore P(-1 \leq X < 1) = P(X = -1) = \frac{1}{4}$$

$$P(1 \leq X < 2) ?$$



$$(1 \leq X < 2) = \{\omega : X(\omega) < 2, X(\omega) \geq 1\}$$

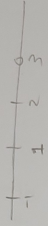
$$= \{\omega : X(\omega) = 1, -1\} = \{\omega : X(\omega) = 1\} \cup \{\omega : X(\omega) = -1\}$$
$$= (X=1) \cup (X=-1)$$

$$P(1 \leq X < 2) = P\{(X=1) \cup (X=-1)\}$$

$$= P(X=1) + P(X=-1)$$

$$= \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

$$P(2 \leq X < 3) ?$$



$$(2 \leq X < 3) = \{\omega : X(\omega) < 3, X(\omega) \geq 2\}$$

$$= \{\omega : X(\omega) = 2, 1, -1\}$$

$$= (X=2) \cup (X=1) \cup (X=-1)$$

$$P(2 \leq X < 3) = P(X=2) + P(X=1) + P(X=-1)$$

$$= \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$P(X \geq 3) = P(\omega : X(\omega) > 3, X(\omega) = 3)$$

$$= 1 - P(X < 3) + P(X=3)$$

$$= 1 - \{P(X=-1) + P(X=2) + P(X=1)\} + P(X=3)$$

$$= 1 - \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{8}\right) + \frac{1}{2}$$

$$= 1 - \left(\frac{1}{4} + \frac{1}{4}\right) + \frac{1}{2}$$

$$= 1 - \frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

(17)