Example :-

1 toss of a fair coin twice , and let X be the no. of heads.

$\Omega = \{HH, HT, TH, TT\}$

$R_X = \{0, 1, 2\}$

$P_X(x) = P(X = x)$

$P_X(0) = P(X=0) = P(TT) = \frac{1}{4}$

$P_X(1) = P(X=1) = P\{TH, HT\} = \frac{2}{4} = \frac{1}{2}$

$P_X(2) = P(X=2) = P\{HH\} = \frac{1}{4}$

$$F_X(x) = P(X \leq x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \leq x < 1 \\ \frac{3}{4} & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

H.w. Show how to find $F_X(x)$?

Discrete random variables and Probability mass function

Let X be a random variable with CDF $F_X(x)$.
If $F_X(x)$ changes value only at discrete set of points and is constant between these points. Then X is a discrete random variable.
Also, if R_X is a finite set of points or a countable, infinite set of points.

Definition:- (Probability mass function)

Let X be a discrete random variable with range $R_X = \{x_1, x_2, x_3, x_4, \dots\}$.
The function:-

$$P_X(x_k) = P(X = x_k) \quad , k = 1, 2, 3, \dots$$

is called the probability mass function (pmf) of X