Test -1-

Answer only $\underline{\underline{3}}$ questions:-

<u>Q1</u> :- Choose only two:-

1-| Prove that If $A_1, A_2, ---, A_n$ are a partition of the Sample space Ω. Then for any event $B \subset \Omega$:-

$$P(A_i | B) = \frac{P(A_i) \times P(B | A_i)}{\sum_{j=1}^{n} P(A_j) \times P(B | A_j)} \quad , i = 1, 2, --- n$$

2-| prove that for any two events A and B :-

$$(A - B)^c = A^c \cup B$$

3-| prove that for any two events A and B :-

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

<u>Q2</u> :-

We roll a pair of fair dice one time. What is the probability that the Sum of the two number. is 2 ? is 7 ? is 11 ?

Q3

An urn contains 3 red balls and 5 blue balls. If we draw a ball at random from the urn without replacing it and then we draw a second ball from the urn. Find the probability that both balls have the same color.

Q4

Urn 1 contains 2 red balls and 4 blue balls, Urn 2 contains 10 red balls and 2 blue balls. If an urn is chosen at random and a ball is drawn from the chosen urn. What is the probability that the selected ball is blue?

Q1

①

Proof:-

$A_1, A_2, \cdots, A_n$ are partition of sample space Ω, then

$\Omega = A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n$ and $A_i \cap A_j = \phi \quad \forall i \neq j$

∵ $B \subseteq \Omega$

∴ $B = B \cap \Omega = B \cap (A_1 \cup A_2 \cup \cdots \cup A_n)$

$= (B \cap A_1) \cup (B \cap A_2) \cup \cdots \cup (B \cap A_n)$

$$P(A_i \mid B) = \frac{P(A_i \cap B)}{P(B)} \qquad —①$$

∵ $P(B) = P\left\{ (B \cap A_1) \cup (B \cap A_2) \cup \cdots \cup (B \cap A_n) \right\}$

$= P(B \cap A_1) + P(B \cap A_2) + \cdots + P(B \cap A_n)$, لأن

$(B \cap A_i) \cap (B \cap A_j) = \phi$

$i \neq j$

$(B \mid A_i) = \frac{P(B \cap A_i)}{P(A_i)}$

$\qquad —②$

∴ $P(B \cap A_i) = P(B \mid A_i) P(A_i) \qquad —③$

نعوض ③ , ⑤ مِن ① نحصل على:

$$P(A_i | B) = \frac{P(A_i) \times P(B|A_i)}{\sum\limits_{j=1}^{n} P(A_j) \times P(B|A_j)}$$

②

A   B
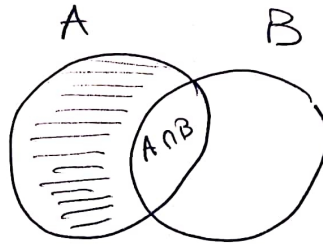
$$A - B = A \cap B^c$$

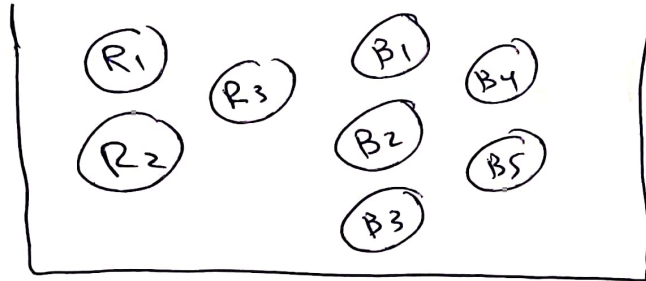$$(A-B)^c = (A \cap B^c)^c = A^c \cup B$$

من قانون دي مورجان

(De Morgan law)

③ موجود برهانه في المحاضرات

Q2

تحليل ضمن المحاضرات

Q3

5 Blue and 3 Red

كناكرة واحد بدون ارجاع و كناكرة ثانية

خان مُضا ولينة φ

$$
\Omega = \left\{
\begin{array}{l}
(R_1, R_2), (R_1, R_3), (R_1, B_1), (R_1, B_2), (R_1, B_3), (R_1, B_4), \\
(R_1, B_5), (R_2, R_1), (R_2, R_3), (R_2, B_1), \\
(R_2, B_2), (R_2, B_3), (R_2, B_4), (R_2, B_5), \\
(R_3, R_1), (R_3, R_2), (R_3, B_1), (R_3, B_2), (R_3, B_3) \\
(R_3, B_4), (R_3, B_5), (B_1, B_2), (B_1, B_3), (B_1, B_4) \\
(B_1, B_5), (B_1, R_1), (B_1, R_2), (B_1, R_3) \\
(B_2, B_1), (B_2, B_3), (B_2, B_4), (B_2, B_5), \\
(B_2, R_1), (B_2, R_2), \quad\quad (B_2, R_3),
\end{array}
\right.
$$

$$\left\{ \begin{array}{l} (B_3, R_1) ,(B_3, R_2), (B_3, R_3), (B_3, B_1), (B_3, B_2), \\ (B_3, B_4), (B_3, B_5), (B_4, B_1), (B_4, B_2), \\ (B_4, B_3), (B_4, B_5), (B_4, R_1), (B_4, R_2), \\ (B_4, R_3), (B_5, B_1), (B_5, B_2), (B_5, B_3), \\ (B_5, B_4), (B_5, R_1), (B_5, R_2)(B_5, R_3) \end{array} \right\}$$
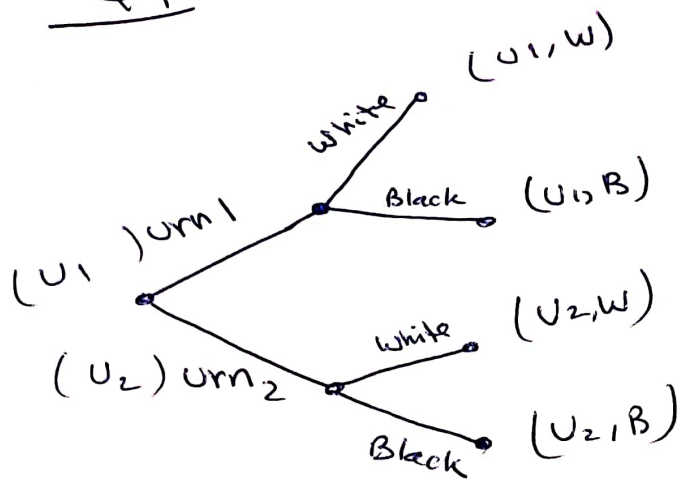
#Ω = 56

A : the two balls have the same colour

$$A = \{(R_1, R_2), (R_1, R_3), (R_2, R_1), (R_2, R_3), (R_3, R_1),$$
$$(R_3, R_2), (B_1, B_2), (B_1, B_3), (B_1, B_4),$$
$$(B_1, B_5), (B_2, B_1), (B_2, B_3), (B_2, B_4),$$
$$(B_2, B_5), (B_3, B_1), (B_3, B_2), (B_3, B_4),$$
$$(B_3, B_5), (B_4, B_1), (B_4, B_2), (B_5, B_1),$$
$$(B_5, B_2), (B_5, B_3), (B_5, B_4), (B_4, B_3), (B_4, B_5)\}$$

$$\#A = 26$$

$$P(A) = \frac{\#A}{\#\Omega} = \frac{26}{56} = \frac{13}{28}$$

Q4

$$\Omega = \left\{ \ (U_1, W) \ , (U_1, B) \ , (U_2, W) \ , (U_2, B) \right\}$$

$$A = \left\{ (U_1, B) \ , (U_2, B) \right\}$$

$$P(A) = P(U_1, B) + P(U_2, B)$$

$$= P(B|U_1) \times P(U_1) + P(B|U_2) \times P(U_2)$$

$$= \frac{4}{6} \times \frac{1}{2} + \frac{2}{12} \times \frac{1}{2}$$

$$= \frac{1}{3} + \frac{1}{12}$$

$$= \frac{4}{12} + \frac{1}{12}$$

$$= \frac{5}{12}$$