Chapter (1) VECTORS

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1.1 Coordinate Systems
1.2 Vector and Scalar Quantities
1.3 Some Properties of Vectors
1.4 Components of a Vector and Unit Vectors

Introduction

Vectors are essential to physics and engineering. Many fundamental physical quantities are vectors, including displacement, velocity, force, and electric and magnetic vector fields. Scalar products of vectors define other fundamental scalar physical quantities, such as energy.
In introductory physics, vectors are Euclidean quantities that have geometric representations as arrows in one dimension (in a line), in two dimensions (in a plane), or in three dimensions (in space). They can be added, subtracted or multiplied. In this chapter, we explore elements of vector algebra for applications in mechanics and in electricity and magnetism. Vector operations also have numerous generalizations in other branches of physics.

1.1 Coordinate Systems

Many aspects of physics involve a description of a location in space. In Chapter 1, for example, we saw that the mathematical description of an object's motion requires a method for describing the object's position at various times. This description is accomplished with the use of coordinates, and in Chapter 1 we used the **Cartesian coordinate system**, in which horizontal and vertical axes intersect at a point defined as the origin (Fig. 1.1). Cartesian coordinates are also called ***rectangular coordinates***.

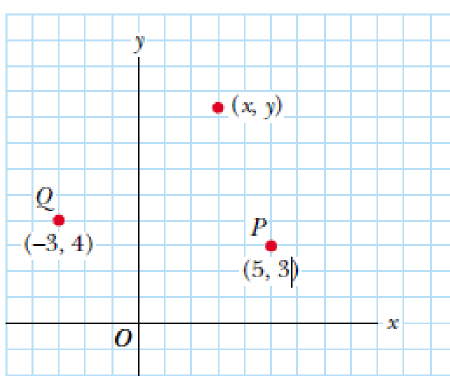
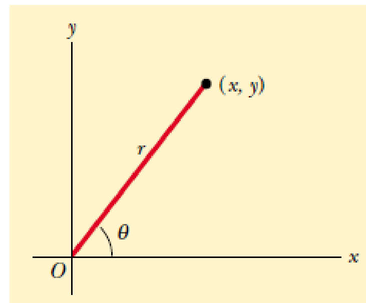


Figure 1.1 Designation of points in a Cartesian coordinate system. Every point is labeled with coordinates (x, y).

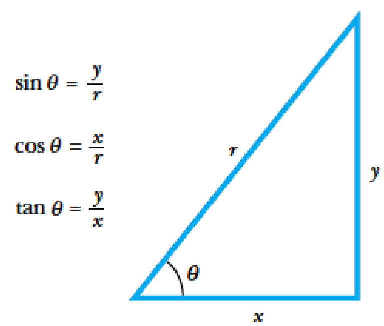
Sometimes it is more convenient to represent a point in a plane by its plane polar coordinates (**r**, **θ**), as shown in Figure1.2a. In this ***polar coordinate system***, r is the distance from the origin to the point having Cartesian coordinates (**x**, **y**), and **θ** is the angle between a line drawn from the origin to the point and a fixed axis. This fixed axis is usually the positive **x** axis, and **θ** is usually measured counter clockwise from it. From the right triangle in Figure 1.2b, we find that **sin θ = y/r** and that **cos θ = x/r**. Therefore, starting with the plane polar coordinates of any point, we can obtain the Cartesian coordinates by using the equations:

$$x = r\cos\theta \qquad (1-1)$$

$$y = r\sin\theta \qquad (1-2)$$



(a)



(b)

Figure **1.2** **(a)** The plane polar coordinates of a point are represented by the distance r and the angle θ, where θ is measured counterclockwise from the positive x axis. **(b)** The right triangle used to relate (x, y) to (r,θ).

Furthermore, the definitions of trigonometry tell us that

$$\tan\theta = \frac{y}{x} \qquad (1-3)$$

$$r = \sqrt{x^2 + y^2} \qquad (1-4)$$

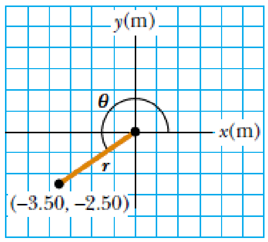
Equation 1.4 is the familiar Pythagorean theorem.

These four expressions relating the coordinates (x, y) to the coordinates (r, θ) apply only when θ is defined as shown in Figure 1.2a—in other words, when positive θ is an angle measured counterclockwise from the positive x axis. (Some scientific calculators perform conversions between Cartesian and polar coordinates based on these standard conventions.) If the reference axis for the polar angle θ is chosen to be one other than the positive x axis or if the sense of increasing θ is chosen differently, then the expressions relating the two sets of coordinates will change.

Example 1.1 Polar Coordinates

The Cartesian coordinates of a point in the *xy* plane are $(x, y) = (-3.50, -2.50)$ m, as shown in Figure 3.3. Find the polar coordinates of this point.

Solution For the examples in this and the next two chapters we will illustrate the use of the General Problem-Solving Strategy outlined at the end of Chapter 2. In subsequent chapters, we will make fewer explicit references to this strategy, as you will have become familiar with it and should be applying it on your own. The drawing in Figure 3.3 helps us to *conceptualize* the problem. We can *categorize* this as a plug-in problem. From Equation 3.4,



Active Figure 3.3 (Example 3.1) Finding polar coordinates when Cartesian coordinates are given.

At the Active Figures link at http://www.pse6.com, you can move the point in the xy plane and see how its Cartesian and polar coordinates change.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2} = 4.30 \text{ m}$$

and from Equation 3.3,

$$\tan\theta = \frac{y}{x} = \frac{-2.50 \text{ m}}{-3.50 \text{ m}} = 0.714$$

$$\theta = 216°$$

Note that you must use the signs of x and y to find that the point lies in the third quadrant of the coordinate system. That is, $\theta = 216°$ and not $35.5°$.

1.2 Vector and Scalar Quantities

As noted in our studies some physical quantities are scalar quantities whereas others are vector quantities. When you want to know the temperature outside so that you will know how to dress, the only information you need is a number and the unit “degrees C” or “degrees F.” Temperature is therefore an example of a scalar quantity:

A scalar quantity is completely specified by a single value with an appropriate unit and has no direction.

Other examples of scalar quantities are volume, mass, speed, and time intervals. The rules of ordinary arithmetic are used to manipulate scalar quantities.

If you are preparing to pilot a small plane and need to know the wind velocity, you must know both the speed of the wind and its direction. Because direction is important for its complete specification, velocity is a vector quantity:

A **vector quantity** is completely specified by a number and appropriate units plus a direction.

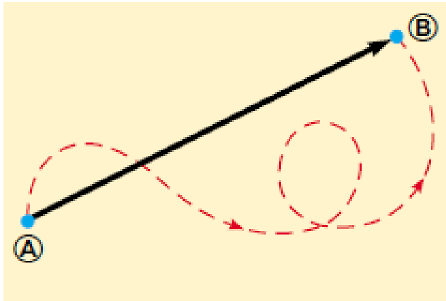


Figure 1.3 As a particle moves from (A) to (B) along an arbitrary path represented by the broken line, its displacement is a vector quantity shown by the arrow drawn from (A) to (B).

Quick Quiz 1.1 Which of the following are vector quantities and which are scalar quantities? (a) your age (b) acceleration (c) velocity (d) speed (e) mass

1.3 Some Properties of Vectors

Equality of Two Vectors

For many purposes, two vectors **A** and **B** may be defined to be equal if they have the same magnitude and point in the same direction. That is, **A** = **B** only if A = B and if **A** and **B** point in the same direction along parallel lines. For example, all the vectors in Figure 1.4 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.

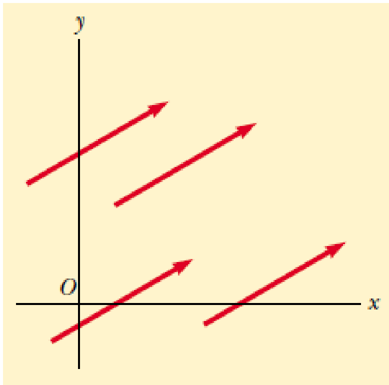


Figure 1.4 These four vectors are equal because they have equal lengths and point in the same direction.

Adding Vectors

The rules for adding vectors are conveniently described by graphical methods. To add vector B to vector A, first draw vector A on graph paper, with its magnitude represented by a convenient length scale, and then draw vector B to the same scale with its tail starting from the tip of A, as shown in Figure 1.5. **The resultant vector R = A + B** is the vector drawn from the tail of A to the tip of B.

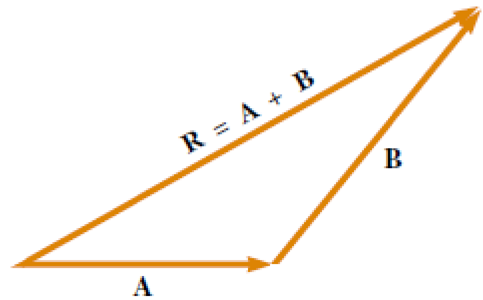


Figure 1.5 When vector **B** is added to vector **A**, the resultant **R** is the vector that runs from the tail of **A** to the tip of **B**.

For example, if you walked 3.0 m toward the east and then 4.0 m toward the north, as shown in Figure 1.6, you would find yourself 5.0 m from where you started, measured at an angle of 53° north of east. Your total displacement is the vector sum of the individual displacements.

A geometric construction can also be used to add more than two vectors. This is shown in Figure 1.7 for the case of four vectors. The resultant vector **R = A + B + C + D** is the vector that completes the polygon. In other words, **R is the vector drawn from the tail of the first vector to the tip of the last vector.**

When two vectors are added, the sum is independent of the order of the addition.

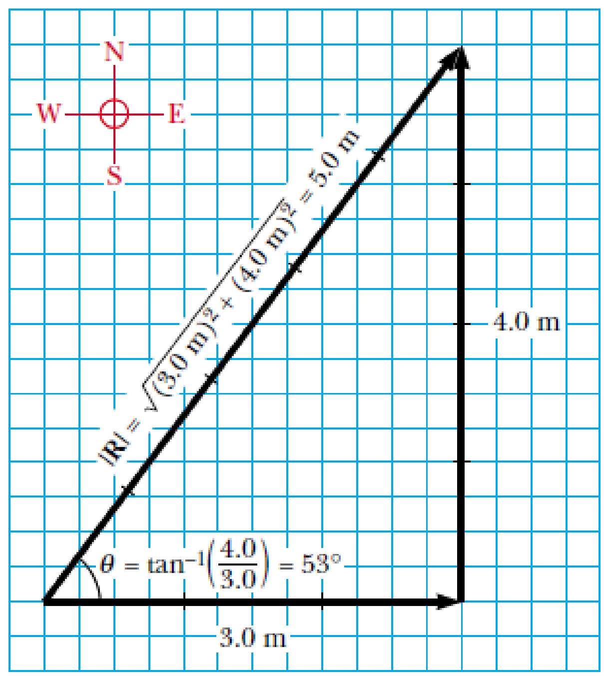


Figure 1.6 Vector addition. Walking first 3.0 m due east and then 4.0m due north leaves you 5.0 m from your starting point.

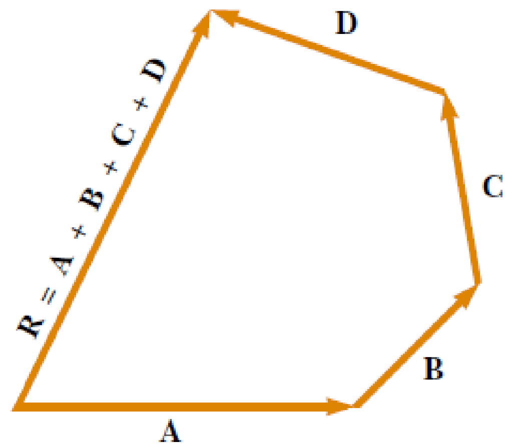


Figure 1.7 Geometric construction for summing four vectors. The resultant vector **R** is by definition the one that completes the polygon.

This can be seen from the geometric construction in Figure 1.8 and is known as the commutative law of addition:

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \qquad (1-5)$$

When three or more vectors are added, their sum is independent of the way in which the individual vectors are grouped together. A geometric proof of this rule for three vectors is given in Figure 1.9. This is called **the associative law of addition**:

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C} \qquad (1-6)$$

In summary, a vector quantity has both magnitude and direction and also obeys the laws of vector addition as described in Figures 1.5 to 1.9. When two or more vectors are added together, all of them must have the same units and all of them must be the same type of quantity. It would be meaningless to add a velocity vector (for example, 60 km/h to the east) to a displacement vector (for example, 200 km to the north) because they represent different physical quantities. The same rule also applies to scalars. For example, it would be meaningless to add time intervals to temperatures.

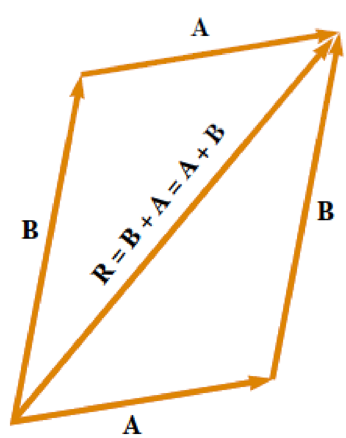


Figure 1.8 This construction shows that **A**+ **B** = **B** + **A** in other words, that vector addition is commutative.

Negative of a Vector

The negative of the vector A is defined as the vector that when added to A gives zero for the vector sum. That is, **A + (-A) = 0**. The vectors **A** and **-A** have the same magnitude but point in opposite directions.

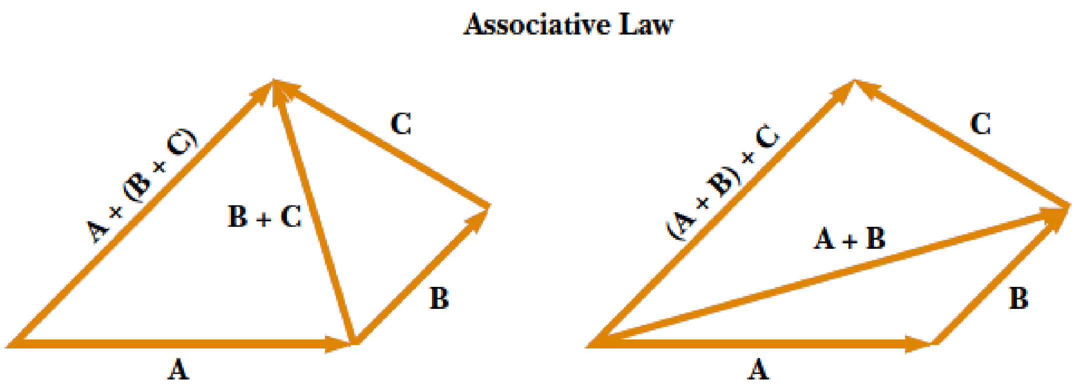


Figure 1.9 Geometric constructions for verifying the associative law of addition.

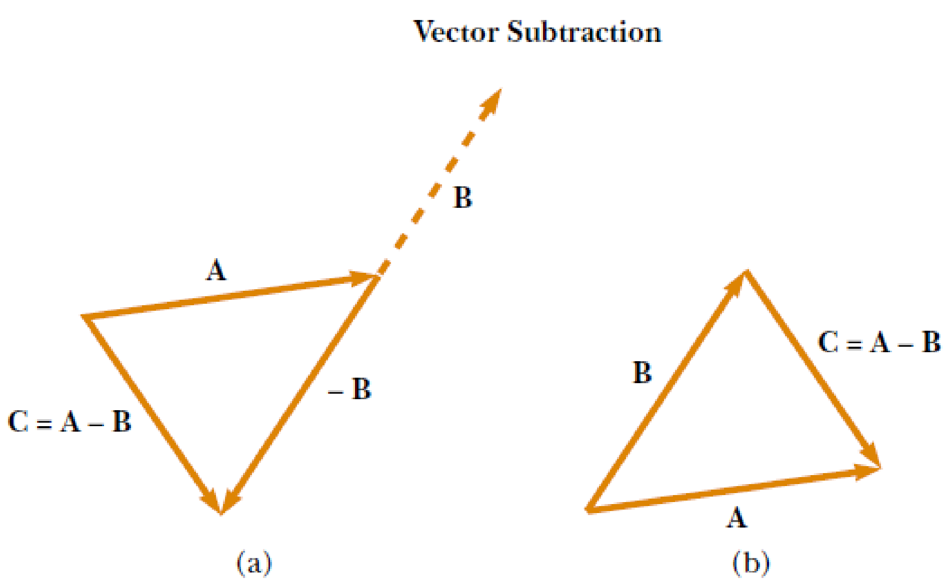


Figure 1.10 (a) This construction shows how to subtract vector **B** from vector **A**. The vector **-B** is equal in magnitude to vector B and points in the opposite direction. To subtract **B** from **A**, apply the rule of vector addition to the combination of A and **-B**: Draw **A** along some convenient axis, place the tail of **-B** at the tip of **A**, and C is the difference **A - B**. (b) A second way of looking at vector subtraction. The difference vector **C = A - B** is the vector that we must add to **B** to obtain **A**.

Subtracting Vectors

The operation of vector subtraction makes use of the definition of the negative of a vector. We define the operation **A - B** as vector **-B** added to vector **A**:

$$\mathbf{A} - \mathbf{B} = \mathbf{A}(-\mathbf{B}) \qquad (1-7)$$

The geometric construction for subtracting two vectors in this way is illustrated in Figure 1.10a. Another way of looking at vector subtraction is to note that the difference **A - B** between two vectors **A** and **B** is what you have to add to the second vector to obtain the first. In this case, the vector **A - B** points from the tip of the second vector to the tip of the first, as Figure 1.10b shows.

Quick Quiz 1.2 The magnitudes of two vectors A and B are A = 12 units and B = 8 units. Which of the following pairs of numbers represents the largest and smallest possible values for the magnitude of the resultant vector R = A + B? (a) 14.4 units, 4 units (b) 12 units, 8 units (c) 20 units, 4 units (d) none of these answers.

Quick Quiz 1.3 If vector B is added to vector A, under what condition does the resultant vector A + B have magnitude A + B? (a) A and B are parallel and in the same direction. (b) A and B are parallel and in opposite directions. (c) A and B are perpendicular.

Quick Quiz 1.4 If vector **B** is added to vector **A**, which *two* of the following choices must be true in order for the resultant vector to be equal to zero? (a) **A** and **B** are parallel and in the same direction. (b) **A** and **B** are parallel and in opposite directions. (c) **A** and **B** have the same magnitude. (d) **A** and **B** are perpendicular.

Example 1.2 **A Vacation Trip**

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north, as shown in Figure 1.11a. Find the magnitude and direction of the car's resultant displacement.

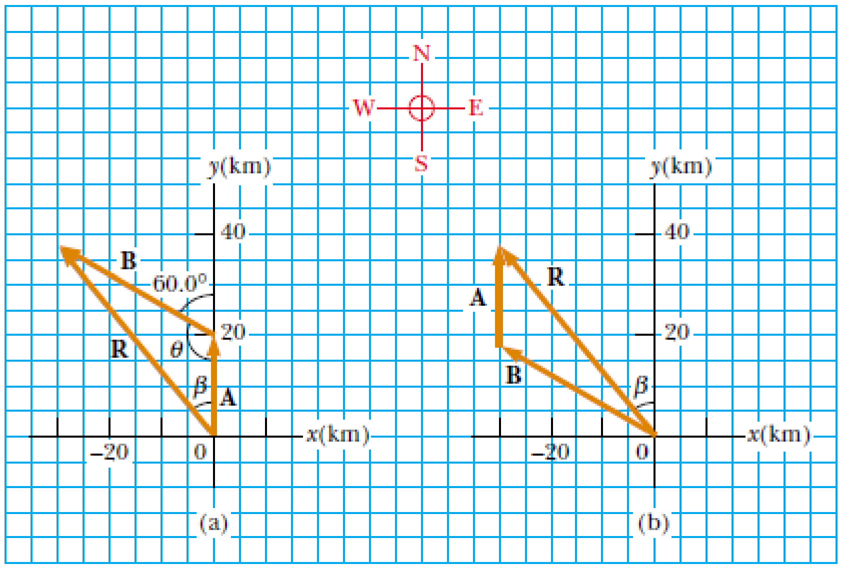


Figure 1.9 (Example 1.2) (a) Graphical method for finding the resultant displacement vector **R** = **A** + **B**. (b) Adding the vectors in reverse order (**B** + **A**) gives the same result for **R**.

Solution

The vectors A and B drawn in Figure 1.11a help us to *conceptualize* the problem. We can *categorize* this as a relatively simple analysis problem in vector addition. The displacement R is the resultant when the two individual displacements A and B are added. We can further categorize this as a problem about the analysis of triangles, so we appeal to our expertise in geometry and trigonometry.

In this example, we show two ways to *analyze* the problem of finding the resultant of two vectors. The first way is to solve the problem geometrically, using graph paper and a protractor to measure the magnitude of R and its direction in Figure 1.11a. (In fact, even when you know you are going to be carrying out a calculation, you should sketch the vectors to check your results.) With an ordinary ruler and protractor, a large diagram typically gives answers to two-digit but not to three-digit precision.

The second way to solve the problem is to analyze it algebraically. The magnitude of R can be obtained from the law of cosines as applied to the triangle. With θ = 180° - 60° = 120° and $R^2 = A^2 + B^2 - 2AB\cos\theta$, we find that

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$
$$= \sqrt{(20.0\text{ km})^2 + (35.0\text{ km})^2 - 2(20.0\text{ km})(35.0\text{ km})\cos 120°}$$
$$= \boxed{48.2\text{ km}}$$

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta = \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\boxed{\beta = 39.0°}$$

Multiplying a Vector by a Scalar

If vector A is multiplied by a positive scalar quantity *m*, then the product *m*A is a vector that has the same direction as A and magnitude *mA*. If vector A is multiplied by a negative scalar quantity -*m*, then the product -*m*A is directed opposite A. For example, the vector 5A is five times as long as A and points in the same direction as A; the vector -1/3A is one-third the length of A and points in the direction opposite A.

1.4 Components of a Vector and Unit Vectors

The graphical method of adding vectors is not recommended whenever high accuracy is required or in three-dimensional problems. In this section, we describe a method of adding vectors that makes use of the projections of vectors along coordinate axes. These projections are called the **components** of the vector. Any vector can be completely described by its components.

Consider a vector **A** lying in the ***xy*** plane and making an arbitrary angle θ with the positive ***x*** axis, as shown in Figure 1.12a. This vector can be expressed as the sum of two other vectors **A*x*** and **A*y***.

From Figure 1.12b, we see that the three vectors form a right triangle and that **A** = **A*x*** + **A*y***. We shall often refer to the "components of a vector A," written A*x* and A*y* (without the boldface notation). The component A*x* represents the projection of **A** along the *x* axis, and the component A*y* represents the projection of **A** along the y axis. These components can be positive or negative. The component A*x* is positive if A*x* points in the positive x direction and is negative if A*x* points in the negative x direction. The same is true for the component Ay.

From Figure 1.12 and the definition of sine and cosine, we see that cos θ = A*x*/A and that sin θ " Ay/A. Hence, the components of **A** are

$$A_x = A\cos\theta \qquad (1-8)$$

$$A_y = A \sin \theta \qquad (1-9)$$

These components form two sides of a right triangle with a hypotenuse of length *A*. Thus, it follows that the magnitude and direction of A are related to its components through the expressions:

$$A = \sqrt{{A_x}^2 + {A_y}^2} \qquad (1-10)$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \qquad (1-11)$$

Note that **the signs of the components A*x* and A*y* depend on the angle θ**. For example, if **θ** = 120°, then A*x* is negative and A*y* is positive. If **θ** = 225°, then both A*x* and A*y* are negative. Figure 1.13 summarizes the signs of the components when A lies in the various quadrants.
When solving problems, you can specify a vector **A** either with its components A*x* and A*y* or with its magnitude and direction A and **θ**.

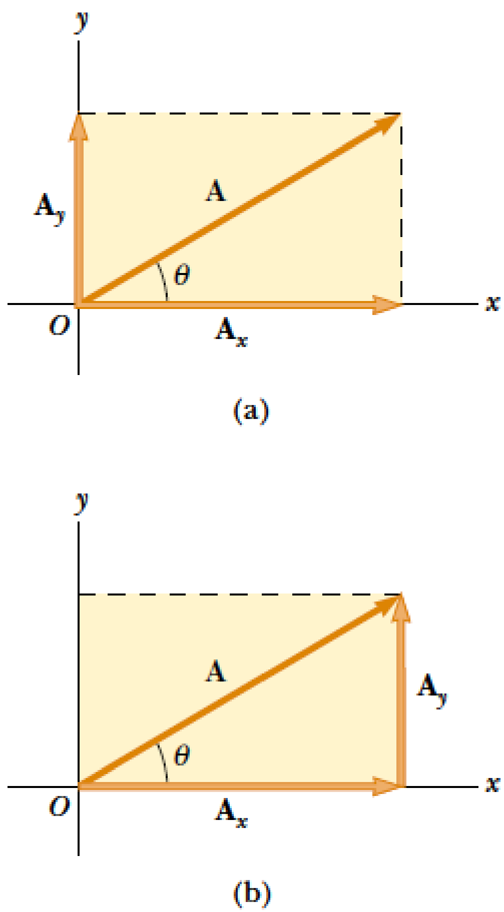


Figure 1.12 (a) A vector **A** lying in the *xy* plane can be represented by its component vectors **A***x* and **A***y* . (b) The *y* component vector **A***y* can be moved to the right so that it adds to **A***x* . The vector sum of the component vectors is **A**. These three vectors form a right triangle.

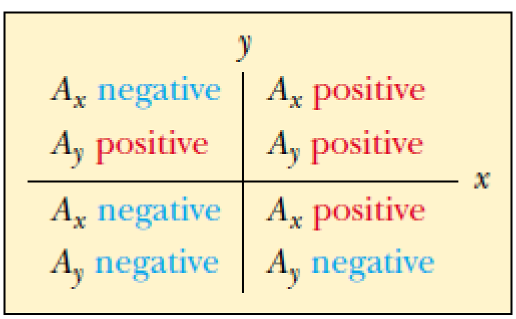
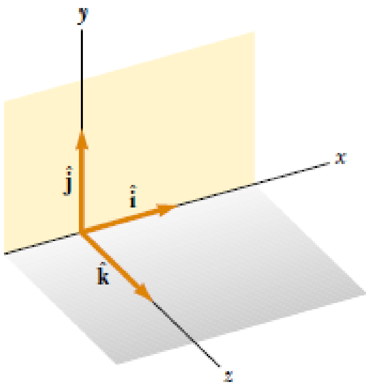


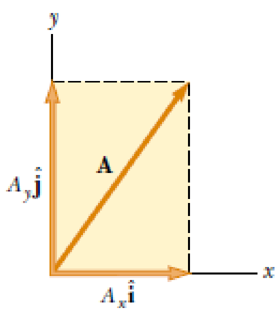
Figure 1.13 The signs of the components of a vector **A** depend on the quadrant in which the vector is located.

Unit Vectors

1



(a)



(b)

Active Figure 1.15 (a) The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\mathbf{k}$ are directed along the x, y, and z axes, respectively. (b) Vector $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$ lying in the xy plane has components A_x and A_y.

Vector quantities often are expressed in terms of unit vectors. **A unit vector is a dimensionless vector having a magnitude of exactly 1.** Unit vectors are used to specify a given direction and have no other physical significance. They are used solely as a convenience in describing a direction in space. We shall use the symbols $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ to represent unit vectors pointing in the positive x, y, and z directions, respectively. (The "hats" on the symbols are a standard notation for unit vectors.) The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ form a set of mutually perpendicular vectors in a right-handed coordinate system, as shown in Figure **1.15a** The magnitude of each unit vector equals 1; that is, $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$.

Consider a vector **A** lying in the xy plane, as shown in Figure **1.15b**. The product of the component A_x and the unit vector $\hat{\mathbf{i}}$ is the vector $A_x\hat{\mathbf{i}}$, which lies on the x axis and has magnitude $|A_x|$. (The vector $A_x\hat{\mathbf{i}}$ is an alternative representation of vector $\mathbf{A}_x$.) Likewise, $A_y\hat{\mathbf{j}}$ is a vector of magnitude $|A_y|$ lying on the y axis. (Again, vector $A_y\hat{\mathbf{j}}$ is an alternative representation of vector $\mathbf{A}_y$.) Thus, the unit–vector notation for the vector **A** is

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \qquad \textbf{(1-12)}$$

For example, consider a point lying in the xy plane and having Cartesian coordinates (x, y), as in Figure 3.17. The point can be specified by the **position vector r**, which in unit–vector form is given by

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} \qquad \textbf{(1.13)}$$

This notation tells us that the components of **r** are the lengths x and y.

Now let us see how to use components to add vectors when the graphical method is not sufficiently accurate. Suppose we wish to add vector **B** to vector **A** in Equation 3.12, where vector **B** has components B_x and B_y. All we do is add the x and y components separately. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is therefore

$$\mathbf{R} = (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}) + (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}})$$

or

$$\mathbf{R} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} \qquad \textbf{(1-14)}$$

Because $\mathbf{R} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$, we see that the components of the resultant vector are

$$R_x = A_x + B_x \qquad \textbf{(1-15)}$$
$$R_y = A_y + B_y$$

We obtain the magnitude of **R** and the angle it makes with the *x* axis from its components, using the relationships

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \quad (1\text{-}16)$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x} \quad (1\text{-}17)$$

We can check this addition by components with a geometric construction, as shown in Figure 3.18. Remember that you must note the signs of the components when using either the algebraic or the graphical method.

At times, we need to consider situations involving motion in three component directions. The extension of our methods to three-dimensional vectors is straightforward. If **A** and **B** both have *x*, *y*, and *z* components, we express them in the form

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}} \quad (1\text{-}18)$$

$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} \quad (1\text{-}19)$$

The sum of **A** and **B** is

$$\mathbf{R} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} + (A_z + B_z)\hat{\mathbf{k}} \quad (1\text{-}20)$$

Note that Equation 1.20 differs from Equation 1.14 in Equation 1-20 the resultant vector also has a *z* component $R_z = A_z + B_z$. If a vector **R** has *x*, *y*, and *z* components, the magnitude of the vector is $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$. The angle θ_x that **R** makes with the *x* axis is found from the expression $\cos\theta_x = R_x/R$, with similar expressions for the angles with respect to the *y* and *z* axes.

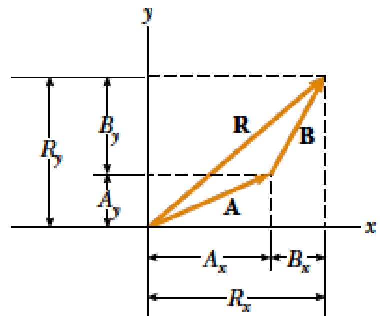


Figure 1.17 This geometric construction for the sum of two vectors shows the relationship between the components of the resultant **R** and the components of the individual vectors.

Example 1.3 The Sum of Two Vectors

Find the sum of two vectors **A** and **B** lying in the *xy* plane and given by

$$\mathbf{A} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ m} \quad \text{and} \quad \mathbf{B} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}})\text{ m}$$

Solution You may wish to draw the vectors to *conceptualize* the situation. We *categorize* this as a simple plug-in problem. Comparing this expression for **A** with the general expression $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$, we see that $A_x = 2.0$ m and $A_y = 2.0$ m. Likewise, $B_x = 2.0$ m and $B_y = -4.0$ m. We obtain the resultant vector **R**, using Equation 1.14

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (2.0 + 2.0)\hat{\mathbf{i}}\text{ m} + (2.0 - 4.0)\hat{\mathbf{j}}\text{ m}$$
$$= (4.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}})\text{ m}$$

or

$$R_x = 4.0\text{ m} \qquad R_y = -2.0\text{ m}$$

The magnitude of **R** is found using Equation 1.16

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0\text{ m})^2 + (-2.0\text{ m})^2} = \sqrt{20}\text{ m}$$
$$= 4.5\text{ m}$$

We can find the direction of **R** from Equation 3.17:

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0\text{ m}}{4.0\text{ m}} = -0.50$$

Your calculator likely gives the answer $-27°$ for $\theta = \tan^{-1}(-0.50)$. This answer is correct if we interpret it to mean 27° clockwise from the *x* axis. Our standard form has been to quote the angles measured counterclockwise from the $+x$ axis, and that angle for this vector is $\theta = 333°$.

Example 1.4 The Resultant Displacement

A particle undergoes three consecutive displacements: $\mathbf{d}_1 = (15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$ cm, $\mathbf{d}_2 = (23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}})$ cm and $\mathbf{d}_3 = (-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$ cm. Find the components of the resultant displacement and its magnitude.

Solution Three-dimensional displacements are more difficult to imagine than those in two dimensions, because the latter can be drawn on paper. For this problem, let us *conceptualize* that you start with your pencil at the origin of a piece of graph paper on which you have drawn x and y axes. Move your pencil 15 cm to the right along the x axis, then 30 cm upward along the y axis, and then 12 cm *vertically away* from the graph paper. This provides the displacement described by $\mathbf{d}_1$. From this point, move your pencil 23 cm to the right parallel to the x axis, 14 cm parallel to the graph paper in the $-y$ direction, and then 5.0 cm vertically downward toward the graph paper. You are now at the displacement from the origin described by $\mathbf{d}_1 + \mathbf{d}_2$. From this point, move your pencil 13 cm to the left in the $-x$ direction, and (finally!) 15 cm parallel to the graph paper along the y axis. Your final position is at a displacement $\mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3$ from the origin.

Despite the difficulty in conceptualizing in three dimensions, we can *categorize* this problem as a plug-in problem due to the careful bookkeeping methods that we have developed for vectors. The mathematical manipulation keeps track of this motion along the three perpendicular axes in an organized, compact way:

$$\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3$$
$$= (15 + 23 - 13)\hat{\mathbf{i}}\text{ cm} + (30 - 14 + 15)\hat{\mathbf{j}}\text{ cm} + (12 - 5.0 + 0)\hat{\mathbf{k}}\text{ cm}$$
$$= (25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}})\text{ cm}$$

The resultant displacement has components $R_x = 25$ cm, $R_y = 31$ cm, and $R_z = 7.0$ cm. Its magnitude is

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$
$$= \sqrt{(25\text{ cm})^2 + (31\text{ cm})^2 + (7.0\text{ cm})^2} = 40\text{ cm}$$

Example 1.5 Taking a Hike

Interactive

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.

Solution We *conceptualize* the problem by drawing a sketch as in Figure 1.18 If we denote the displacement vectors on the first and second days by **A** and **B**, respectively, and use the car as the origin of coordinates, we obtain the vectors shown in Figure 1.18 Drawing the resultant **R**, we can now *categorize* this as a problem we've solved before—an addition of two vectors. This should give you a hint of the power of categorization—many new problems are very similar to problems that we have already solved if we are careful to conceptualize them.

We will *analyze* this problem by using our new knowledge of vector components. Displacement **A** has a magnitude of 25.0 km and is directed 45.0° below the positive x axis. From Equations 3.8 and 3.9, its components are

$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = 17.7\text{ km}$$

$$A_y = A\sin(-45.0°) = (25.0\text{ km})(-0.707) = -17.7\text{ km}$$

The negative value of A_y indicates that the hiker walks in the negative y direction on the first day. The signs of A_x and A_y also are evident from Figure 1.18

The second displacement **B** has a magnitude of 40.0 km and is 60.0° north of east. Its components are

$$B_x = B\cos 60.0° = (40.0\text{ km})(0.500) = \boxed{20.0\text{ km}}$$

$$B_y = B\sin 60.0° = (40.0\text{ km})(0.866) = \boxed{34.6\text{ km}}$$

(B) Determine the components of the hiker's resultant displacement **R** for the trip. Find an expression for **R** in terms of unit vectors.

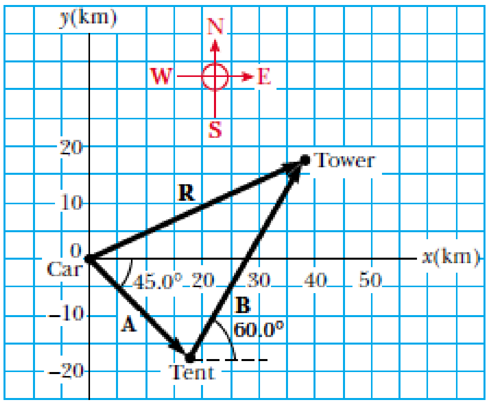


Figure 1.18 (Example 1.5 The total displacement of the hiker is the vector **R** = **A** + **B**.

Solution The resultant displacement for the trip **R** = **A** + **B** has components given by Equation 1.15

$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = \boxed{37.7\text{ km}}$$

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = \boxed{16.9\text{ km}}$$

In unit–vector form, we can write the total displacement as

$$\mathbf{R} = \boxed{(37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}})\text{ km}}$$

Using Equations 1.16 and 1.17 we find that the vector **R** has a magnitude of 41.3 km and is directed 24.1° north of east.

Let us *finalize.* The units of **R** are km, which is reasonable for a displacement. Looking at the graphical representation in Figure 1.18 we estimate that the final position of the hiker is at about (38 km, 17 km) which is consistent with the components of **R** in our final result. Also, both components of **R** are positive, putting the final position in the first quadrant of the coordinate system, which is also consistent with Figure 1.18