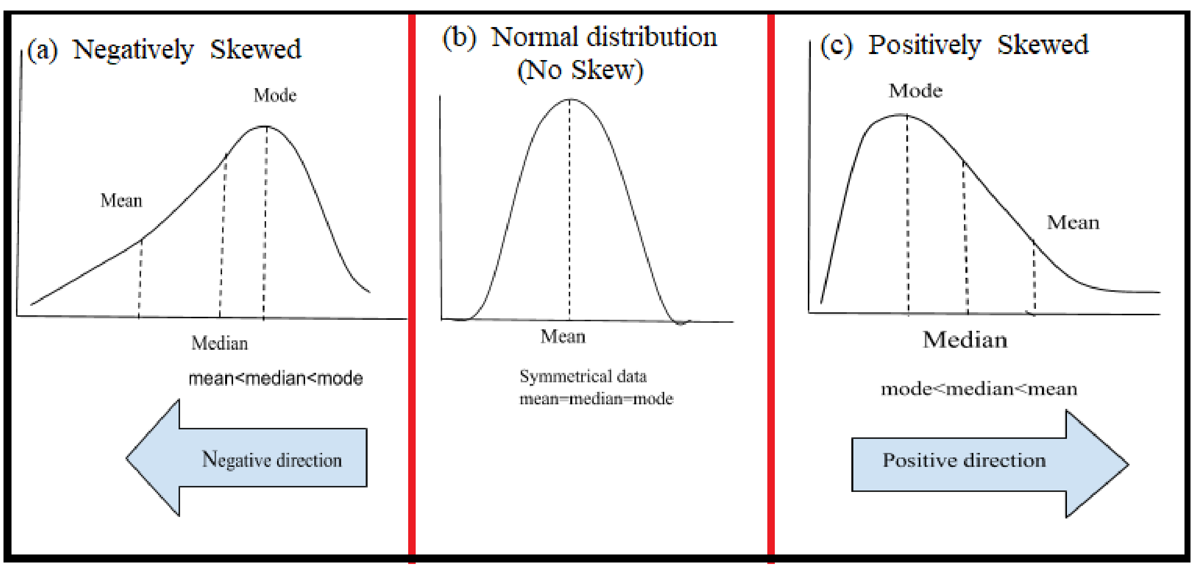
# Measures of Shape: Skewness and Kurtosis

The measure of central tendency and measure of dispersion can describe the distribution but they are not sufficient to describe the nature of the distribution. For this purpose, we use other two statistical measures that compare the shape to the normal curve called Skewness and Kurtosis.
Skewness and Kurtosis are the two important characteristics of distribution that are studied in descriptive statistics

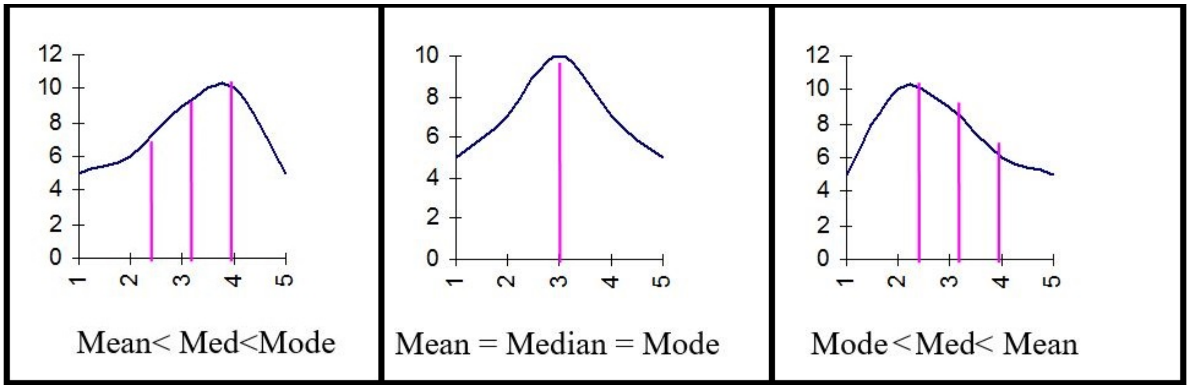
## 1-Skewness

Skewness is a statistical number that tells us if a distribution is symmetric or not. A distribution is symmetric if the right side of the distribution is similar to the left side of the distribution.
If a distribution is symmetric, then the Skewness value is 0.
i.e. If a distribution is Symmetric (normal distribution): median= mean= mode, (Skewness value is 0)
If Skewness is greater than 0, then it is called right-skewed or that the right tail is longer than the left tail. If Skewness is less than 0, then it is called left-skewed or that the left tail is longer than the right tail.

For example, the symmetrical and skewed distributions are shown by curves as:



And other example




The Formula of Skewness is:

$$\text{Skewness} = \frac{\sum (x - \bar{x})^3}{(n - 1) \cdot S^3}$$

Where:
S: standard deviation
$\bar{X}$ : Mean

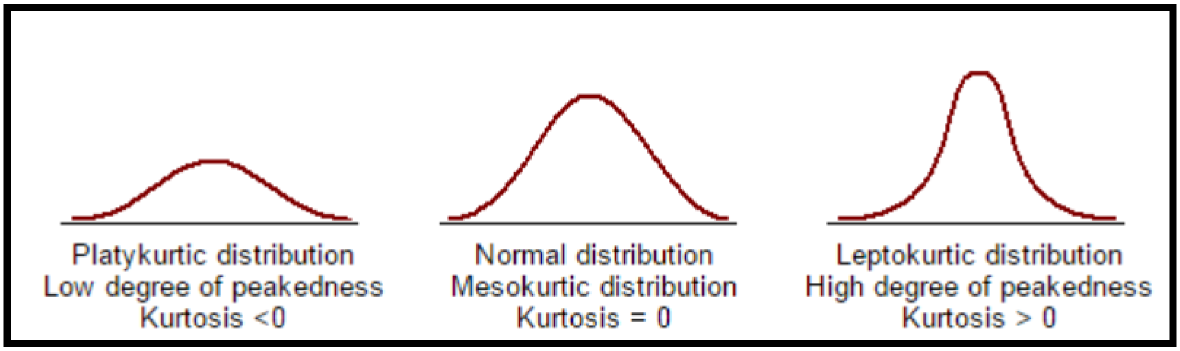
## 2-Kurtosis

Kurtosis is a statistical number that tells us if a distribution is taller or shorter than a normal distribution. If a distribution is similar to the normal distribution, the Kurtosis value is 0. If Kurtosis is greater than 0, then it has a higher peak compared to the normal distribution. If Kurtosis is less than 0, then it is flatter than a normal distribution.

There are three types of distributions:

Leptokurtic: Sharply peaked with fat tails, and less variable.
Mesokurtic: Medium peaked
Platykurtic: Flattest peak and highly dispersed.

For example, The different types of Kurtosis:




The Formula of kurtosis is:

$$\text{Kurtosis} = \frac{\sum (x - \bar{x})^4}{(n - 1) \cdot S^4}$$

Where:

$S$: standard deviation $\bar{X}$ : Mean

**Examples: Calculate Sample Skewness and Sample Kurtosis from the following grouped data**

| Class | Frequency |
|---|---|
| 2 - 4 | 3 |
| 4 - 6 | 4 |
| 6 - 8 | 2 |
| 8 - 10 | 1 |

**Solution**:

| Classes | Mid value ($x$) | $f$ | $f \cdot x$ | $(x-\bar{x})$ | $f \cdot (x-\bar{x})^2$ | $f \cdot (x-\bar{x})^3$ | $f \cdot (x-\bar{x})^4$ |
|---|---|---|---|---|---|---|---|
| **2 - 4** | **3** | **3** | **3×3= 9** | **3-5.2=-2.2** | **3×-2.2×-2.2=14.52** | **14.52×-2.2= -31.944** | **70.27** |
| **4 - 6** | **5** | **4** | **4×5= 20** | **5-5.2=-0.2** | **4×-0.2×-0.2=0.16** | **0.16×-0.2= -0.032** | **0.0064** |
| **6 - 8** | **7** | **2** | **2×7= 14** | **7-5.2=1.8** | **2×1.8×1.8=6.48** | **6.48×1.8=11.664** | **20.98** |
| **8 - 10** | **9** | **1** | **1×9= 9** | **9-5.2=3.8** | **1×3.8×3.8=14.44** | **14.44×3.8= 54.872** | **208.5** |
| --- | --- | --- | --- | --- | --- | --- | |
| **-TOTAL-** | **--** | **$n$=10** | **$\sum f \cdot x$=52** | **`--** | **=35.6** | **=34.56** | **=299.79** |

Mean = $\sum \boldsymbol{f} \cdot \boldsymbol{x} = \frac{\sum \boldsymbol{f} \cdot \boldsymbol{x}}{\sum f} = \frac{52}{10}$ =5.2

Calculate Standard deviation (S.D)

$$S.D = \sqrt{\frac{\sum_{i=1}^{n} fi(Xi - \bar{X})^2}{\sum_{i=1}^{n} fi}}$$

$$S.D = \sqrt{\frac{35.6}{10}} = 1.88$$

Calculate the Skewness

$$\boxed{\text{Skewness} = \frac{\sum (x - \bar{x})^3}{(n - 1) \cdot S^3}}$$

$$\text{Skewness} = \frac{34.56}{9*(1.88)^3} = 0.48$$

Calculate the Kurtosis:

$$\boxed{\text{Kurtosis} = \frac{\sum (x - \bar{x})^4}{(n - 1) \cdot S^4}}$$

$$\text{Kurtosis} = \frac{299.79}{9*(1.88)^4} = 2.12$$

**Key Differences Between Skewness and Kurtosis**

This is the fundamental differences between skewness and kurtosis:

1- The characteristic of a frequency distribution that ascertains its symmetry about the mean is called skewness. On the other hand, Kurtosis means the relative pointedness of the standard bell curve, defined by the frequency distribution.

2- Skewness is a measure of the degree of lopsidedness in the frequency distribution. Conversely, kurtosis is a measure of degree of tailedness in the frequency distribution.

3- Skewness is an indicator of lack of symmetry, i.e. both left and right sides of the curve are unequal, with respect to the central point. As against this, kurtosis is a measure of data, that is either peaked or flat, with respect to the probability distribution.

4- Skewness shows how much and in which direction, the values deviate from the mean? In contrast, kurtosis explain how tall and sharp the central peak is.