الجامعة المستنصرية
كلية العلوم
قسم علوم الحاسوب

Computer Graphics

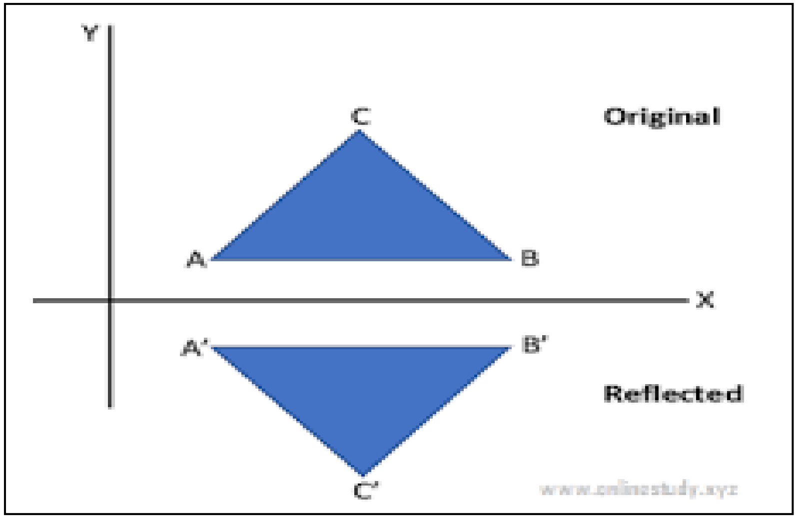
اعداد
أ.م. صلاح طه علاوي

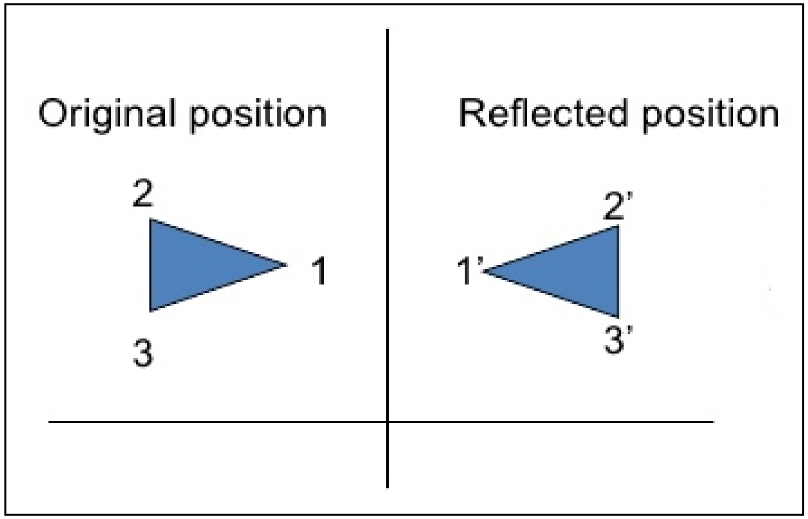
المرحلة الثالثة
مسائي

محاضرة رقم : ٤

Two Dimension Transformation

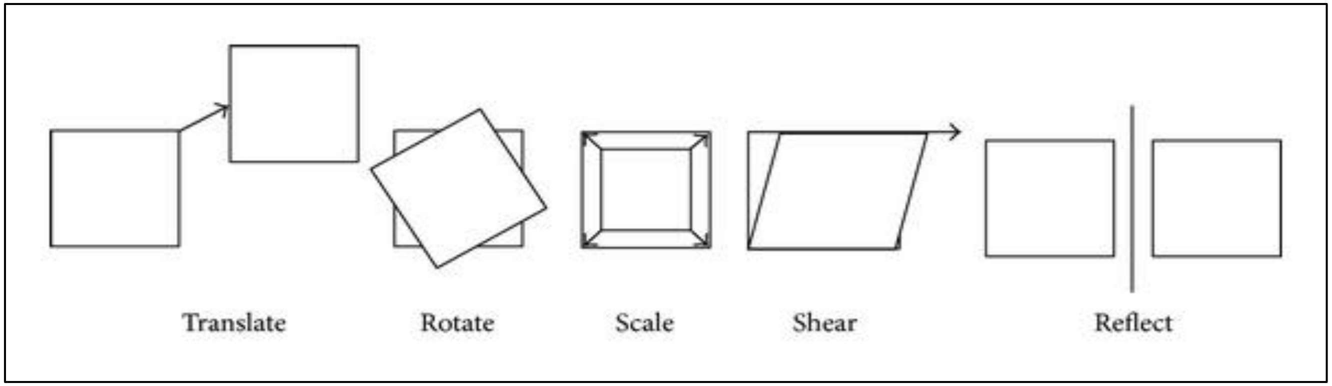
Reflection



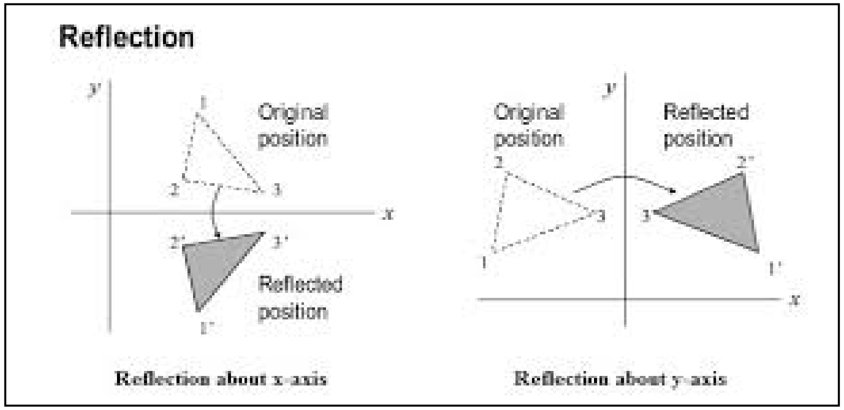


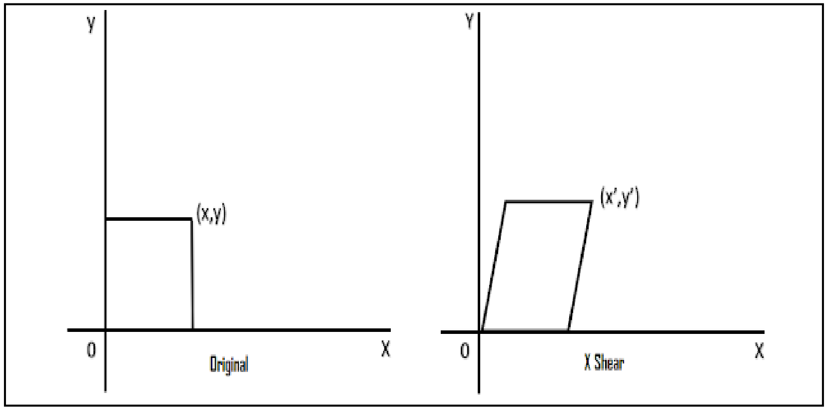
Reflection Transformations

- The three basic transformations of scaling, rotating, and translating are the most useful and most common.



- There are some other transformations which are useful in certain applications. Two such transformations are reflection and shear.

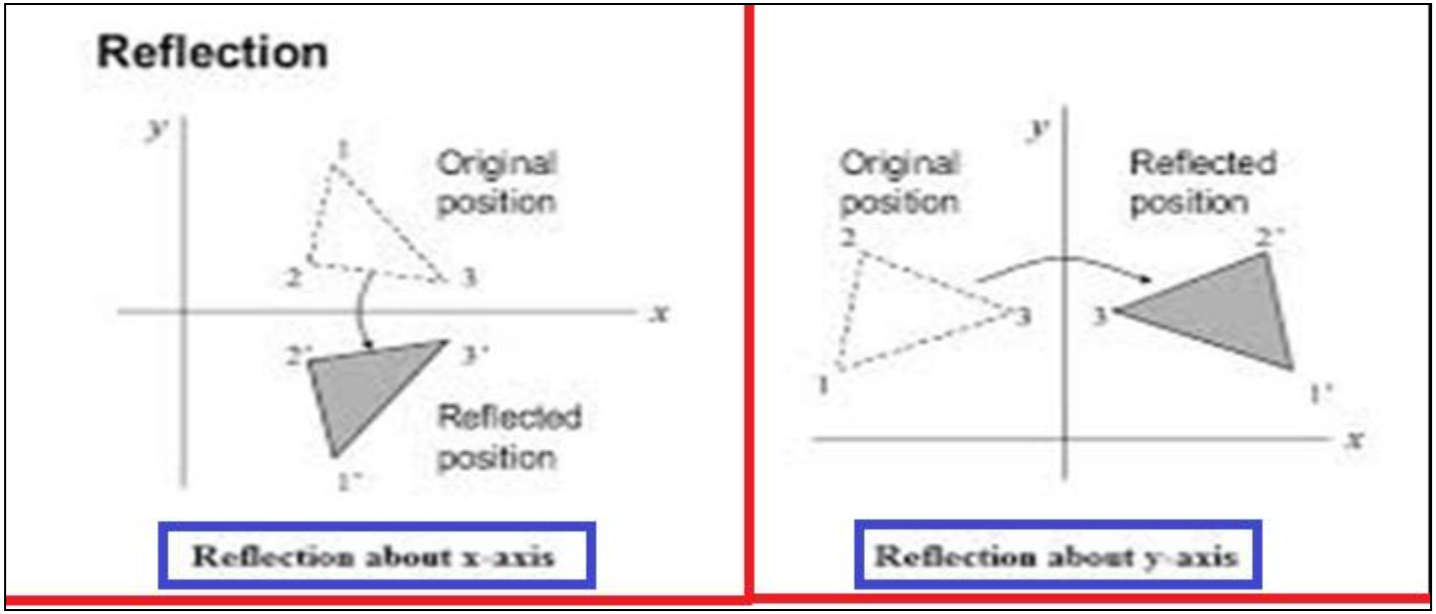




Reflection

Reflections

A reflection is a transformation that produces a *mirror image* of an *object relative* to an axis of reflection. We can choose an axis of reflection in the x-y plane or perpendicular to the x-y plane. Figure below gives example of the reflection in y-direction and in x-direction .



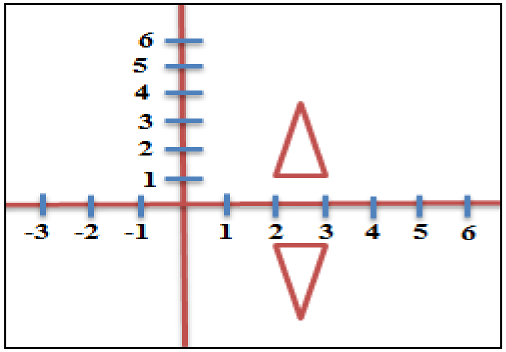
Reflection

1- Reflection on the X-axis

$$X_{new} = X$$
$$Y_{new} = -Y$$

OR

1	0	0
0	-1	0
0	0	1

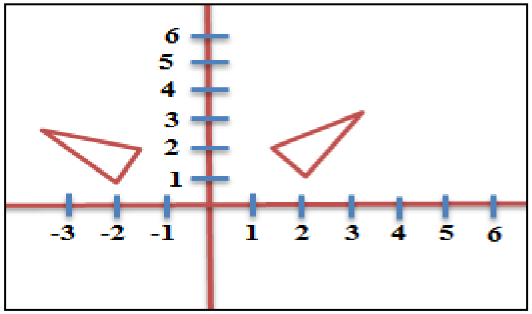


2- Reflection on the Y-axis

$$X_{new} = -X$$
$$Y_{new} = Y$$

OR

-1	0	0
0	1	0
0	0	1

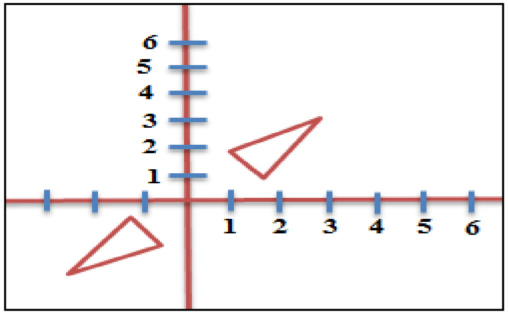


3- Reflection on the origin

$$X_{new} = -X$$
$$Y_{new} = -Y$$

OR

-1	0	0
0	-1	0
0	0	1



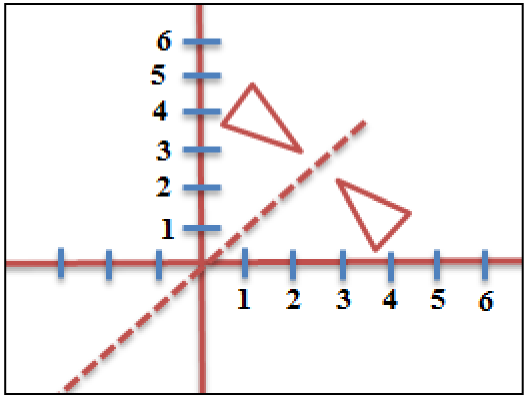
Reflection

4- Reflection on the line Y = X

$X_{new} = Y$
$Y_{new} = X$

OR

0	1	0
1	0	0
0	0	1

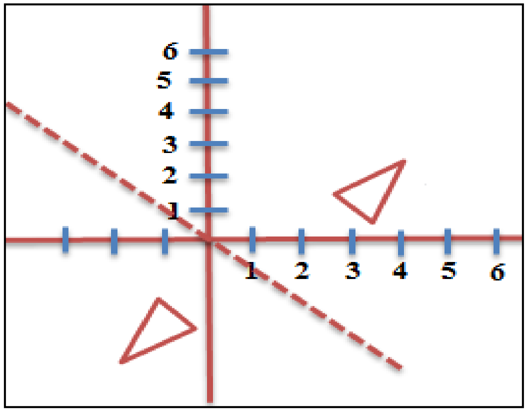


5- Reflection on the line Y= -X

$X_{new} = -Y$
$Y_{new} = -X$

OR

0	-1	0
-1	0	0
0	0	1



Example 1:

Reflect the point P(3,2) in : A- X axis; B- Y axis; C- origin; D-line Y=X;

Solution:

A- X axis

$$\begin{bmatrix} X1\ new & Y1\ new & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \end{bmatrix}$$

B- Y axis

$$\begin{bmatrix} X1\ new & Y1\ new & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 1 \end{bmatrix}$$

C- Origin

$$\begin{bmatrix} X1\ new & Y1\ new & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -2 & 1 \end{bmatrix}$$

D- Line Y=X

$$\begin{bmatrix} X1\ new & Y1\ new & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}$$

Example 2 :

Reflect the triangle with vertices at A(2,4), B(4,6), C(2,6) in : A- X axis

Solution 1:

$X_{new} = X$

$Y_{new} = -Y$

Point (2,4)

$X1_{new} = 2$

$Y1_{new} = -4$

Point (4,6)

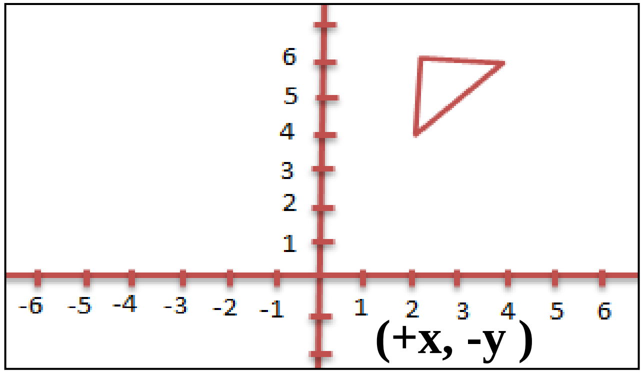
$X1_{new} = 4$

$Y1_{new} = -6$

Point (2,6)

$X1_{new} = 2$

$Y1_{new} = -6$



Solution 2:

X1 new	Y1 new	1
X2 new	Y2 new	1
X3 new	Y3 new	1

=

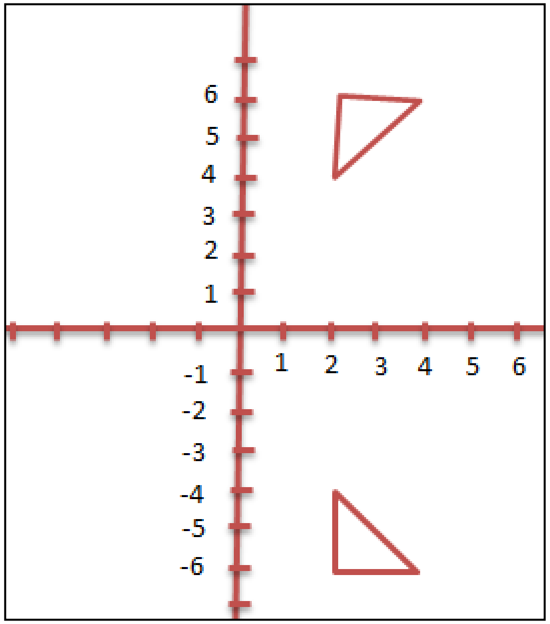
2	4	1
4	6	1
2	6	1

*

1	0	0
0	-1	0
0	0	1

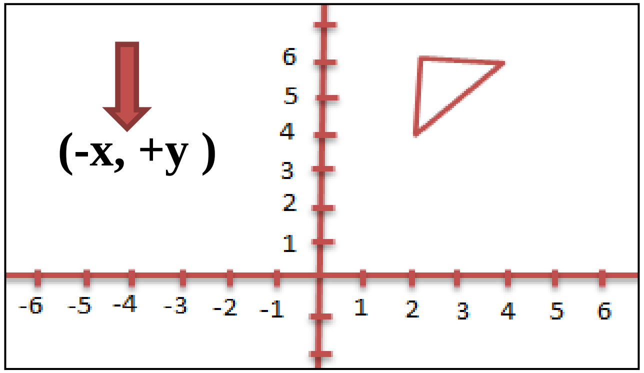
=

2	-4	1
4	-6	1
2	-6	1



Example 3 :

Reflect the triangle with vertices at A(2,4), B(4,6), C(2,6) in : Y axis;



Solution 1:

$X_{new} = -X$

$Y_{new} = Y$

Point (2,4)

$X1_{new} = -2$

$Y1_{new} = 4$

Point (4,6)

$X1_{new} = -4$

$Y1_{new} = 6$

Point (2,6)

$X1_{new} = -2$

$Y1_{new} = 6$

Solution 2:

X1 new	Y1 new	1
X2 new	Y2 new	1
X3 new	Y3 new	1

=

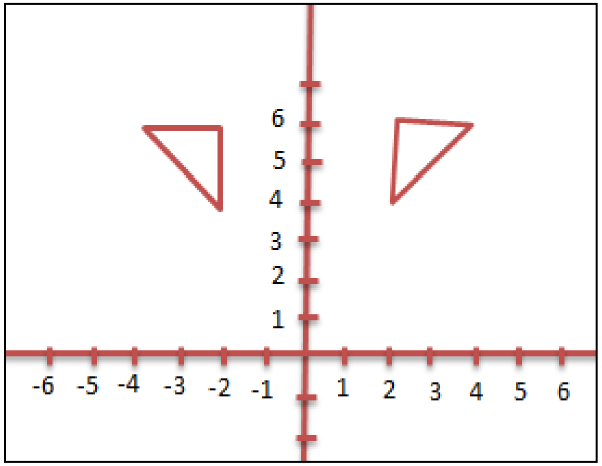
2	4	1
4	6	1
2	6	1

*

-1	0	0
0	1	0
0	0	1

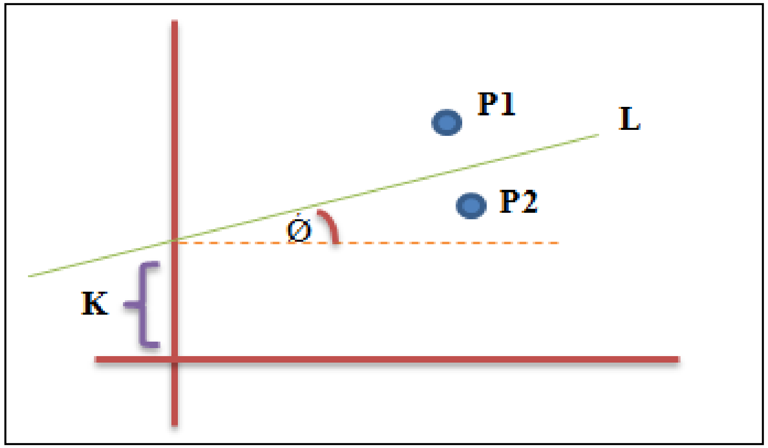
=

-2	4	1
-4	6	1
-2	6	1



Reflection on an arbitrary line

To reflect an object on a line that does not pass through the origin, which is the general case:



As shown in the figure, let the line L intercept with Y axis in the point (0,K) and have an angle of inclination Ǿ degree with respect to the positive direction of X axis . To reflect the point P1 on the line L, we follow the following steps:

1. Move all the points up or down (in the direction of Y axis) so that L pass through the origin

T =

1	0	0
0	1	0
0	- k	1

2. Rotate all the points through (-Ǿ) degree about the origin making L lie along the X axis

R =

Cos θ	-Sin θ	0
Sin θ	Cos θ	0
0	0	1

3. Reflect the point P1 on the X axis

R efX=

1	0	0
0	-1	0
0	0	1

4. Rotate back the points by (-Ǿ) degree so that L back to its original orientation

R_{-1} =

Cos θ	Sin θ	0
-Sin θ	Cos θ	0
0	0	1

5. Shift in the direction of Y axis so that L is back in its original position

T_{-1} =

1	0	0
0	1	0
0	k	1

The sequence of matrices needed to perform this non-standard reflection is:

$$S = T * R * RefX * R_{-1} * T_{-1}$$

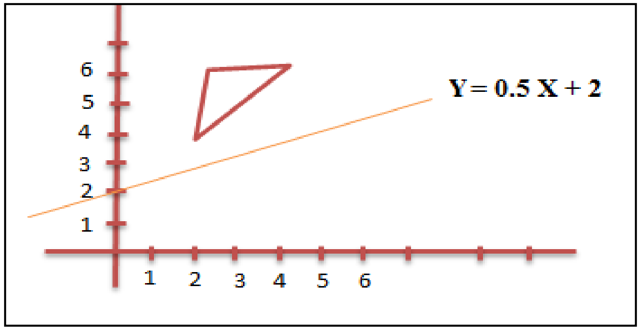
S =

Cos 2Ǿ	Sin 2Ǿ	0
Sin 2Ǿ	-Cos 2Ǿ	0
-K Sin 2Ǿ	K+K Cos 2Ǿ	1

Example 4 :

Find the single matrix that causes all the points in the plane to be reflected in the line with equation Y=0.5X+2 , then apply this matrix to reflect the triangle with vertices at A(2,4), B(4,6), C(2,6) in the line.

Solution:



- **The Cartesian equation of a line in 2D is *Y= M * X +b* where b is the intersection of the lint with the Y axis and M is gradient of the line *M= ΔY / ΔX = Tan Ǿ***

- **So the line *Y=0.5 X + 2* has gradient *M= 0.5* and intersect with the Y axis at the point where *y=2***

- **So K=2 , Tan Ǿ = 0.5 ==> Ǿ = 26.57**
2Ǿ=53.13 , Cos 2Ǿ = 0.6 , Sin 2Ǿ = 0.8

S =

0.6	0.8	0
0.8	- 0.6	0
- 1.6	3.2	1

To reflected the triangle on the line:

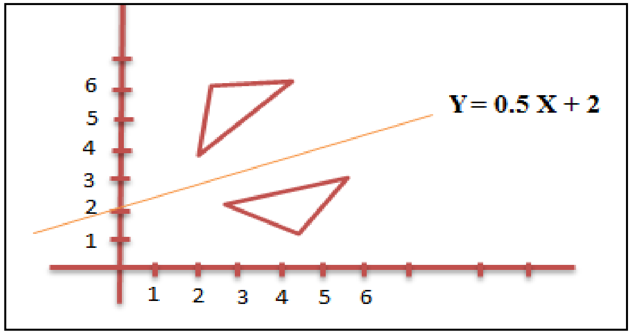
2	4	1
4	6	1
2	6	1

*

0.6	0.8	0
0.8	- 0.6	0
- 1.6	3.2	1

=

2.8	2.4	1
5.6	2.8	1
4.4	1.2	1



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