Lecture 1

The Pressure Gradient Force

1.1 The Pressure Gradient Force

We consider an infinitesimal volume element of air, $\delta V = \delta x \delta y \delta z$, centered at the point x_0, y_0, z_0 as illustrated in Fig. 1.1. Due to random molecular motions, momentum is continually imparted to the walls of the volume element by the surrounding air. This momentum transfer per unit time per unit area is just the pressure exerted on the walls of the volume element by the surrounding air. If the pressure at the center of the volume element is designated by p_0, then the pressure on the wall labeled A in Fig. 1.1 can be expressed in a Taylor series expansion as,

$$p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2} + higher\ order\ terms$$

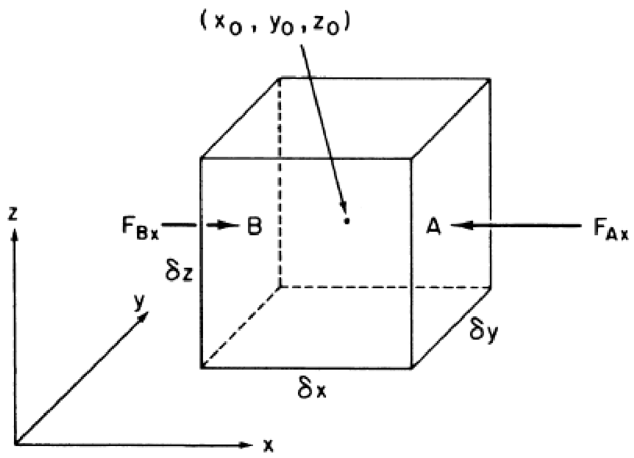


Fig. 1.1 The x component of the pressure gradient force acting on a fluid element.

Neglecting the higher order terms in this expansion, the pressure force acting on the volume element at wall A is

$$F_{AX} = -(p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2})\ \delta x\ \delta z$$

where $\delta y \delta z$ is the area of wall A.

Similarly, the pressure force acting on the volume element at wall B is just

$$F_{BX} = +(p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2})\ \delta y\ \delta z$$

Therefore, the net x component of this force acting on the volume is

$$F_X = F_{AX} + F_{BX} = -\frac{\partial p}{\partial x}\delta x\ \delta y\ \delta z$$

Because the net force is proportional to the derivative of pressure in the direction of the force, it is referred to as the pressure gradient force. The mass m of the differential volume element is simply the density ρ times the volume: $m = \rho\delta x\delta y\delta z$. Thus, the x component of the pressure gradient force per unit mass is

$$\frac{F_X}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

Similarly, it can easily be shown that the y and z components of the pressure gradient force per unit mass are

$$\frac{F_y}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad and \qquad \frac{F_z}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

so that the total pressure gradient force per unit mass is

$$\frac{\vec{F}}{m} = -\frac{1}{\rho}\nabla p \qquad (1.1)$$

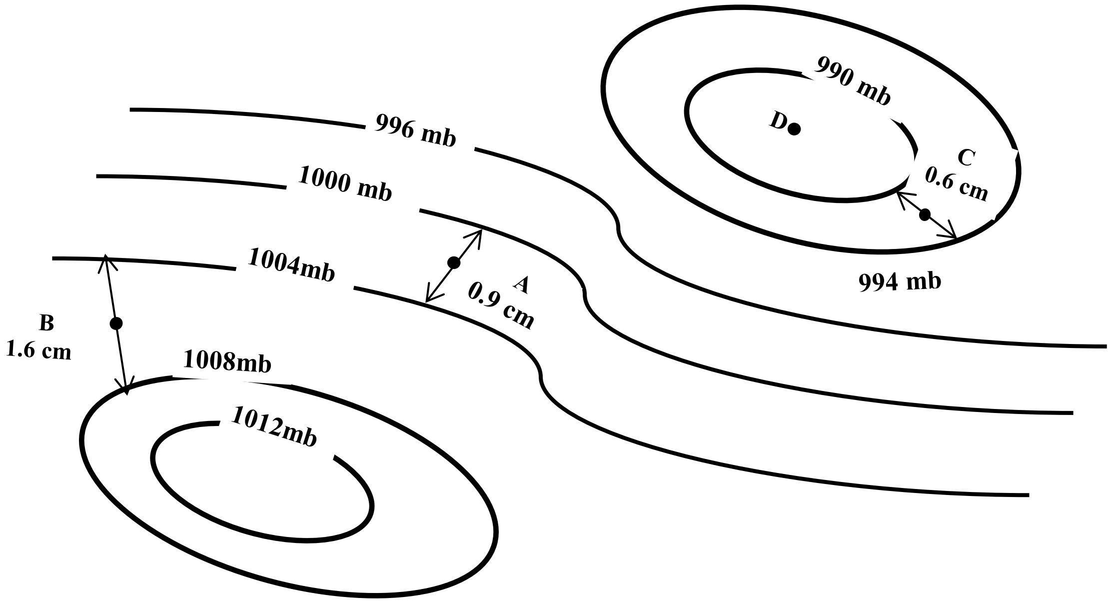
$$\vec{f} = -\frac{1}{\rho}\nabla p \qquad (1.2)$$

It is important to note that this force is proportional to the gradient of the pressure field, not to the pressure itself.

The direction of the pressure gradient force $\vec{f}$ is in the opposite direction of the pressure gradient ∇p , and $\nabla p \approx \frac{\Delta p}{\Delta n}$ where Δp is the contour interval for the isobars and Δn is the horizontal distance between the isobars.

7.2 Example:

At the four points shown in the picture below, estimate the magnitude of the acceleration due to the pressure gradient force. Assume the density is 1.23 kg/m³. The isobars are labeled in mb, scale map is 1 cm/100 km.



$$\vec{f} = -\frac{1}{\rho}\nabla p$$

$$\left|\vec{f}\right| = \frac{1}{\rho}|\nabla p| = \frac{1}{\rho}\frac{\Delta p}{\Delta n}$$

<u>At point B</u>

$$\Delta p = 1008 - 1004 = 4\,mb = 4\,hPa$$

$$\Delta p = 4 \times 10^2\ Pa = 4 \times 10^2 N/m^2$$

$$\Delta n = 1.6\,cm \times \frac{100\,km}{1\,cm} = 160\,km = 160\ \times 10^3\ m$$

$$\left|\vec{f}\right| = \frac{1}{1.23}\ \frac{4\times10^2}{160\ \times10^3} = 0.002\,m/sec^2$$

<u>At point A</u>

$$\Delta p = 1004 - 1000 = 4\,mb = 4 \times 10^2 N/m^2$$

$$\Delta n = 0.9\,cm \times \frac{100\,km}{1\,cm} = 90\,km$$

$$\left|\vec{f}\right| = \frac{1}{1.23}\ \frac{4\times10^2}{90\ \times10^3} = 0.0036\,m/sec^2$$

<u>At point C</u>

$$\Delta p = 994 - 990 = 4\ mb = 4 \times 10^2 N/m^2$$

$$\Delta n = 0.6\ cm \times \frac{100\ km}{1\ cm} = 60\ km$$

$$\left|\vec{f}\right| = \frac{1}{1.23}\ \frac{4\times10^2}{60\times10^3} = 0.0054\ m/sec^2$$

<u>At point D</u>

$$\left|\vec{f}\right| = 0$$

Because $\Delta p = 0$