Lecture 2

The Coriolis Force

2.1 The Coriolis Force

- ✓ The Coriolis force is another **apparent** force, which occur because of the rotation of Earth when viewed from the coordinates that rotate with the earth itself, not from the inertial coordinates.
- ✓ If a person 1 rolls a ball toward person 2 on a frictionless surface at a uniform speed in a straight line. It will appear (for person 1) to deflect to the right (see figure 2.1) {in a direction opposite the rotation}.

  To a person on the ground, the ball appears to travel in straight line (non-accelerating), but from the coordinate system of merry-go-round, the ball accelerates (velocity changes).

Figure 2.1 merry-go-round

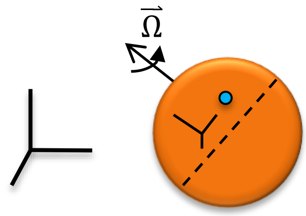
- ✓ The Coriolis force acts perpendicular to the direction of the velocity vector and in direct proportion to its magnitude.
- ✓ The victorial formula of Coriolis force is:

$$\vec{F}_{cor} = -2\vec{\Omega} \times \vec{V} \qquad (2.1)$$

- ✓ In N.H., $\vec{\Omega}$ is counterclockwise, the Coriolis force acts to the right of the velocity vector.
- ✓ The Coriolis force changes only the direction of the particle's motion, not its speed, because the force is always perpendicular to the direction of the motion (just like the centripetal force in uniform circular motion).

2.2 Components of Coriolis Force

- ✓ Consider a parcel of air (unit mass) initially is at rest in a rotating frame. At this moment the centrifugal force is acting along with other forces.
- ✓ Suppose now that the parcel is suddenly set in motion toward the east. The parcel is now rotating faster than the earth, and thus the centrifugal force will be stronger.



- ✓ The total centrifugal force is then composed of that from earth's rotation plus that due to the eastward motion, i.e.:

$$(\Omega + \frac{u}{R})^2\vec{R} = \Omega^2\vec{R} + \frac{2\Omega u}{R}\vec{R} + \frac{u^2}{R^2}\vec{R} \qquad (2.2)$$

where u is the eastward speed, $\vec{R}$ is the position vector from the axis of rotation.

- For synoptic scale motion, $/u/ \ll \Omega R \quad or \quad \frac{/u/}{R} \ll \Omega$
- The last term can be neglected:

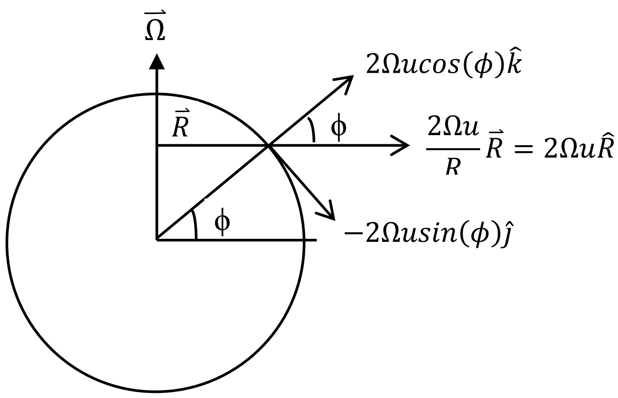
$$\frac{2\Omega u}{R}\vec{R} + \frac{u^2}{R^2}\vec{R} = \frac{2\Omega u}{R}\vec{R}\left(1 + \frac{u}{2\Omega R}\right) = \frac{2\Omega u}{R}\vec{R}$$

The result is the Coriolis force due to the motion along a latitude circle.

- Let's look at the component of this vector, we see that due to the Coriolis force alone:

$$\left(\frac{dv}{dt}\right)_{cor} = -2\Omega u\, sin(\phi)$$

$$\left(\frac{dw}{dt}\right)_{cor} = 2\Omega u\, cos(\phi)$$

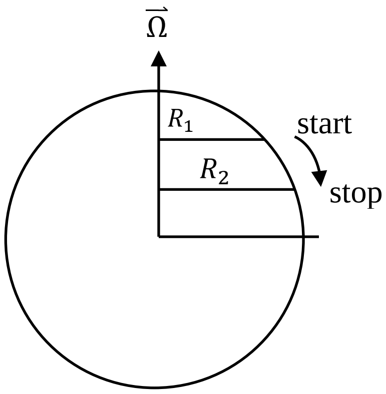


That means if a parcel moves to the east in the horizontal plane in N.H., it deflects southerly and upward; the vice versa for a parcel moves from the east to the west.

If you use numbers, you will find that the vertical component of Coriolis force is much less than the gravity, hence it is not causes large change in the vertical location of the parcel, but the horizontal components could be large in comparison to the horizontal forces (horizontal pressure gradient force is another dominant force).

✓ Now let's repeat this exercise for a parcel displaced toward the equator. What's different here?

- R is the distance from the parcel from the axis of earth rotation changes (increases).



Because the forces acting on the parcel are central, the torque is zero-there is no torque in the east-west direction. Therefore, the parcel <u>conserves angular momentum</u>. So if R increases what will happen to the velocity?

Initially, it has no motion tangential to the direction of rotation. The angular momentum is ΩR^2, so as R increases, the Ω of the parcel has to decrease → the absolute eastward velocity decreases, the parcel starts to deflect toward west. So, the particle moving south is deflected to the west.