Introduction:

The light emitted by an ordinary light source is not an infinitely long, simple harmonic wave but is composed of a jumble of finite wave trains. We therefore call a real monochromatic source as a quasi-monochromatic source. The wave trains issuing out of a quasi-monochromatic source are as shown in figure

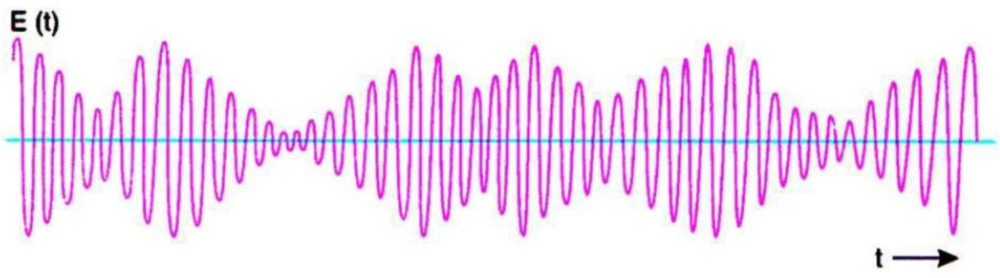

Fig.1

Waves train:

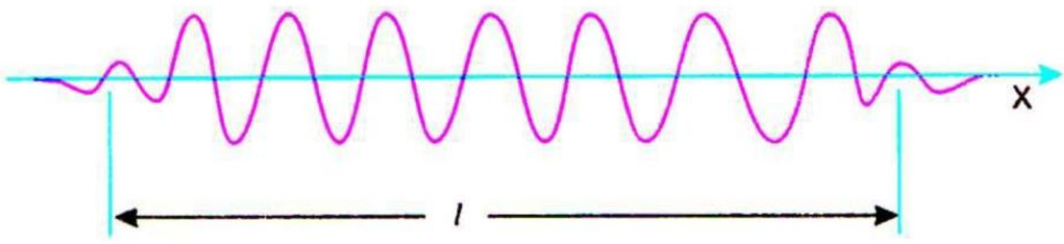

Fig.2: Shows a wave train generated by an atom.

If such a wave train lasts for a time interval Δt, then the length of the wave train in a vacuum is

$$l = c \Delta t \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where c is the velocity of light in a vacuum. For example, if $\Delta t = 10^{-8}$s. and $c = 3 \times 10^{8}\ \mathrm{m/s}$, then $l = (3 \times 10^{8}\ \mathrm{m/s})(10^{-8}\mathrm{s}) = 3\mathrm{m}$.

The number of oscillations present in the wave train is

$$N = \frac{l}{\lambda} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where λ is the wavelength. If we assume $\lambda = 5000\text{A}^{\circ} = 5 \times 10^{-7}\text{m}$, then $\quad N = \frac{3\text{m}}{5\times10^{-7}\text{m}} = 6 \times 10^{6}$.

Thus, a wave train contains about a million wave oscillations in it.

Adding together the wave packets generated by all atoms in the light source, one finds a succession of wave trains, as shown in figure.

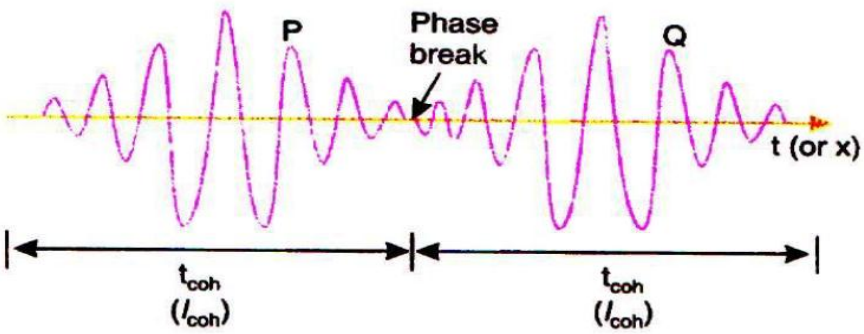

Fig.3

In passing from one wave train to the next, there is an abrupt change in the phase and also in plane of polarization. It is not possible to relate the phase at a point in wave train Q to a point in wave train P.

Consequently there is no correlation between the phase different wave trains. Each wave train has a sustained phase for only about 10^{-8}s, after which a new wave train is emitted with a totally random phase which also lasts only for about 10^{-8}s. The phase of the wave train from one atom will remain constant with respect to the phase of the wave train from another atom for utmost 10^{-8}s. It means that the wave trains can be coherent for a maximum 10^{-8}s only. If two light waves overlap, sustained interference is not observed since the phase relationship between the waves changes rapidly, nearly at the rate of 10^{-8} times per second.

Coherence length and coherence time:

$l_{coh} = c \bigtriangleup t$ …………………. (3)

It is the time, $\bigtriangleup t$, during which the phase of the wave train does not become randomized but undergoes change in a regular systematic way. Coherence time is denoted by t_{coh}. We can therefor write.

$t_{coh} = \bigtriangleup t$ …………………. (4)

$\therefore l_{coh} = c\, t_{coh}$ …………………. (5)

A wave train consists of a group of waves, which have a continuous spread of wavelengths over a finite range $\bigtriangleup \lambda_\circ$ centered on a wavelength $\lambda_\circ$. According to Fourier analysis the frequency bandwidth $\bigtriangleup \nu$ is given by

$$\bigtriangleup \nu = \frac{1}{\bigtriangleup t}$$

Where $\bigtriangleup t$ is the average lifetime of the excited state of the atom. However, $\bigtriangleup t$ is time during which a wave train is radiated by atom and corresponds to the coherence time, t_{coh} , of the wave train.

$$\therefore \bigtriangleup \nu = \frac{1}{\bigtriangleup t} = \frac{1}{t_{coh}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(6)$$

Using the relation (5) in to equation (6), we get

$$\bigtriangleup \nu = \frac{c}{l_{coh}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(7)$$

Bandwidth:

A wave packet is not harmonic wave. Therefore, it cannot be represented mathematically by simple sin functions. The mathematical representation of a wave packet is done in terms of Fourier integral. If light emitted from a source is analyzed with help of a spectrograph, it is known to be made up of discrete spectral lines. Wave packets emitted by atoms form these spectral lines. Therefore, a spectral lines and wave packet are equivalent descriptions. The wavelength of a wave packet or a spectral line is not precisely defined. There is a continuous spread of wavelengths over a finite range, $\Delta\lambda$, centered on a wavelength $\lambda_\circ$. The maximum intensity of the wave packet occurs at $\lambda_\circ$ and the intensity falls off rapidly on either side of $\lambda_\circ$, as shown in figure.

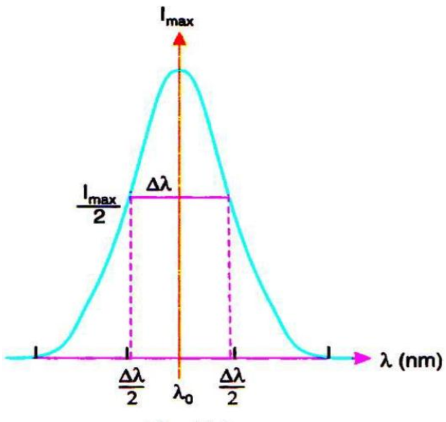

Fig.4

The spread of wavelengths is called the bandwidth. The bandwidth is the wavelength interval from$\lambda_\circ - \Delta\lambda/2$ to $\lambda_\circ + \Delta\lambda/2$ which contains the major portion of the energy of the wave packet. In practice a source, which is said to produce line spectrum, produce a number of sharp wavelength distributions.

Relation between coherence length and bandwidth:

The frequency and wavelength of a light wave are related through the equation

$$\nu = \frac{c}{\lambda} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(8)$$

Where $\lambda_\circ$ is the vacuum wavelength.

Differentiating equation (8) on both sides, we get

$$\Delta\nu = -\frac{c}{\lambda^2}\,\Delta\lambda \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(9)$$

Using the relation (7) into equation (9), we obtain

$$\therefore \frac{c}{l_{coh}} = -\frac{c}{\lambda^2}\,\Delta\lambda$$

Rearranging the terms, we get

$$l_{coh} = \frac{\lambda^2}{\Delta\ \lambda} \qquad \dots\dots\dots\dots\dots\dots\dots..(10)$$

The minus sign has no significance and hence is ignored. Equation (10) means that the coherence length (the length of the wave packet) and the bandwidth of the wave packet are related to each other. The longer the wave packet, the narrower will be the bandwidth. In the limiting case, when the wave is infinitely long, we obtain monochromatic radiation of frequency $\nu_\circ$ (wavelength $\lambda_\circ$).

Form equation (2), the coherence length may be defined as product of the number of wave oscillations N contained in the wave train and of the wavelength, λ. Thus,

$$l_{coh} = N\lambda \qquad \dots\dots\dots\dots\dots\dots\dots\dots..(11)$$

Equation (10) and (11), we get

$$N = \frac{\lambda}{\Delta\ \lambda}$$

$$\therefore \frac{1}{N} = \frac{\Delta\ \lambda}{\lambda} \qquad \dots\dots\dots\dots\dots\dots\dots..(12)$$

Equation (12) shows that the large the number of wave oscillations in a wave packet, the smaller is the bandwidth. In the limiting case, when N is infinitely large, that is when the wave packet is infinitely long; the wave will be monochromatic having a precisely defined wavelength. The dependence of bandwidth on the length of the wave packet is schematically shown in figure.

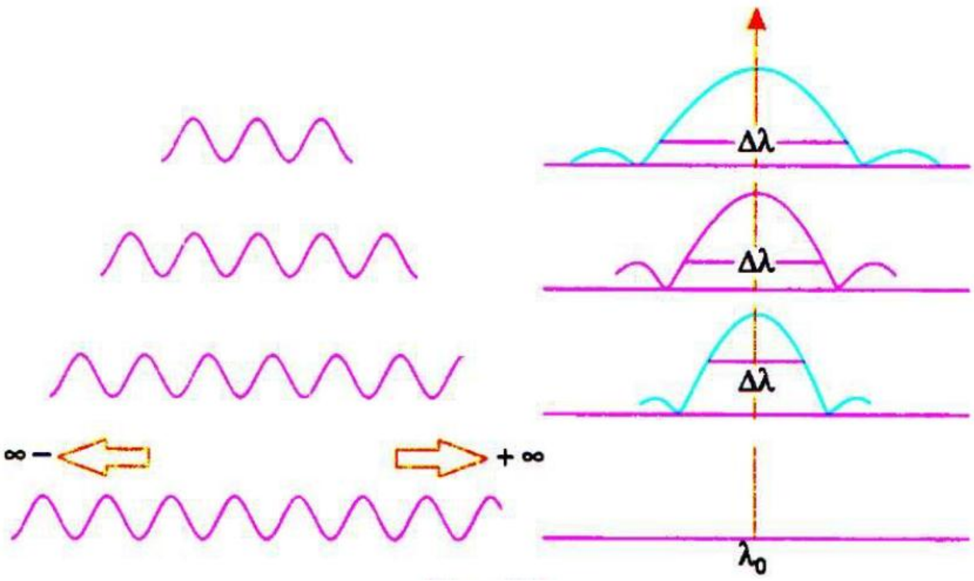

Fig.5

Determination of Coherence Length:

The coherence length can be measured by means of Michelson interferometer. In a Michelson interferometer, a light beam from the source S is incident on a semi-silvered glass plate G (see Fig. 7) and gets divided into two components: one component is reflected, 1 .and the other,2, is transmitted. These two beams, 1 and 2, are reflected back at mirrors M_1 and M_2 respectively and are received by the telescope where interference fringes are produced. It is obvious that the beams produce stationary interference only if they are coherent.

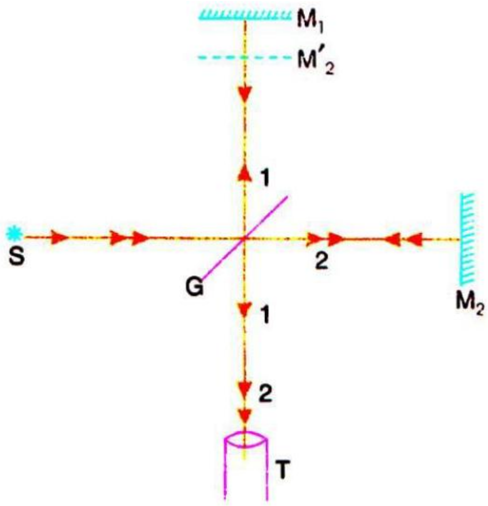

Fig.7

Let M_2' be the image of M_2 formed by G. If the reflecting surfaces M_1 and M_2' (the image of M_2) are separated by a distance d, then 2d will be the path difference between the interfering waves. The condition of fixed phase relationship between the two waves, 1 and 2, will be satisfied if

$$2d \ll l_{coh}$$

In such a case distinct interference fringes will be seen. If, however,

$$2d \gg l_{coh}$$

then the phases of the two waves are not correlated and interference fringes will not be seen. To determine the coherence length of waves emitted by a light source, the distance d between the mirrors M_1 and M_2' (the image of M_2) is varied by moving one of the mirrors. As the distance varies, the contrast of the fringes decreases and ultimately they disappear. The path difference 2d at the particular stage where the fringes disappear gives us the coherence length.

The light from a sodium lamp has coherence length of the order of 1 mm, that of green mercury line is about 1 cm, neon red line 3 cm, red cadmium line 30 cm, orange krypton line 80 cm and that of a commercial He-Ne laser is about 15m. The coherence length of light from some of the lasers goes up to a few km.