Probability

The Concept of Probability

In environmental science and many other subjects, there is certainly no shortage of uncertainty, for example, in our knowledge about whether it will rain or not tomorrow. Uncertainty about such events arises naturally from errors and gaps in measurements, incomplete and incorrect knowledge of the underlying mechanisms, and also from the overall complexity of all the possible interactions in real-world systems.

We try to describe this uncertainty qualitatively by using words such as \likely", \probably", \chance", etc.. However, to make progress scenically it is necessary to use a more quantitative definition of uncertainty.

In 1812, the French mathematician Pierre Simon Laplace defined the word \probability" to mean a number lying between 0 and 1 that measures the amount of certainty for an event to occur. A probability of 1 means the event is completely certain to occur, whereas a probability of 0 means that the event will certainly never occur.

Probability is essential for understanding how samples of data can be drawn from the underlying population, and for making inferences about this population based on sample statistics.

some patterns (spatial distributions or temporal patterns) occur as a result of either totally deterministic or totally random processes. For example, the location of the tornado "touch down" points within a region is the result of random meteorological processes. Sometimes the size of the study area affects the degree of randomness. For example, the number of tornadoes next year within Kansas can be estimated as a probability.

Given that most spatial and temporal patterns are produced by processes that have some degree of uncertainty, meteorologists need to understand and use probability for solving problems. For example, every location on the earth's surface receives a variable amount of precipitation. These data can be recorded over time and across space, and precipitation patterns summarized with calculations such as the mean and standard deviation. However, since precipitation results from complex atmospheric processes, its prediction can only be stated in terms of probabilities, not exact certainty.

Therefore, meteorologists make statements such as "50 percent of the time snowfall in January exceeds 20 inches" or "9 years out of 10, at least 5 inches of rainfall in June." Although probabilities can be stated, the exact amount of snowfall next January or the exact amount of rainfall next June cannot be predicted with certainty.

The study of probability focuses on the occurrence of an event, which can usually result in one of several possible outcomes. Once all possible outcomes have been considered for the event, probability represents the likelihood of a given result or the chance that any outcome actually takes place.

Probability is the likelihood or chance of an event occurring.

$$\text{Probability} = \frac{\text{The number of ways of achieving success}}{\text{The total number of possible outcomes}}$$

Probability can also be thought of as relative frequency (the ratio between the absolute frequency for an outcome and the frequency of all outcomes):

$$\text{Probability} = \frac{\text{Fa}}{\text{F}_E}$$

where:

P, =probability of outcome

Fa. = absolute frequency of outcome

F_E = absolute frequency of all outcomes for event E

Probabilities can also be interpreted as percentages when the denominator of the equation is converted to 100.

For example,

the probability of flipping a coin and it being heads is ½, because there is 1 way of getting a head and the total number of possible outcomes is 2 (a head or tail).

We write P(heads) = ½ .

Example

What is the likelihood of rolling a 6 on a die?

In this problem, the event studied is the roll of the die, and the possible outcomes are the 6 sides of the die.

Since each outcome or side has an equal chance of occurring, the likelihood of rolling a 6 is one out of six or 0.167.

The probability of a 6 (0.167) can be interpreted as an outcome that occurs 16.7 percent of the time.

Example

There are 6 beads in a bag, 3 are red, 2 are yellow and 1 is blue. What is the probability of picking a yellow?

The probability is the number of yellows in the bag divided by the total number of balls, i.e. 2/6 = 1/3.

Example

There is a bag full of colored balls, red, blue, green and orange. Balls are picked out and replaced. John did this 1000 times and obtained the following results:

Number of blue balls picked out: 300

Number of red balls: 200

Number of green balls: 450

Number of orange balls: 50

What is the probability of picking a green ball?

For every 1000 balls picked out, 450 are green.

$$\text{Probability} = \frac{\text{The number of ways of achieving success}}{\text{The total number of possible outcomes}}$$

$$\text{Therefore } P(\text{green}) = \frac{450}{1000} = 0.45$$

Example

Suppose a day is classified as wet (w) if measurable precipitation (0.01 inch or more) falls during the 24-hour period. The day is termed dry if measurable precipitation does not occur. By keeping a record of wet and dry days over a 100-day period, precipitation frequencies can be determined and probabilities calculated from the data

In this example, 62 days Frequency of Dry are categorized as dry and 38 as wet.

a) The probability of a wet day occurring (P,) is:

$$\text{Probability} = \frac{\text{The number of wet days}}{\text{The total days}}$$

$$P = \frac{38}{100} = 0.38$$

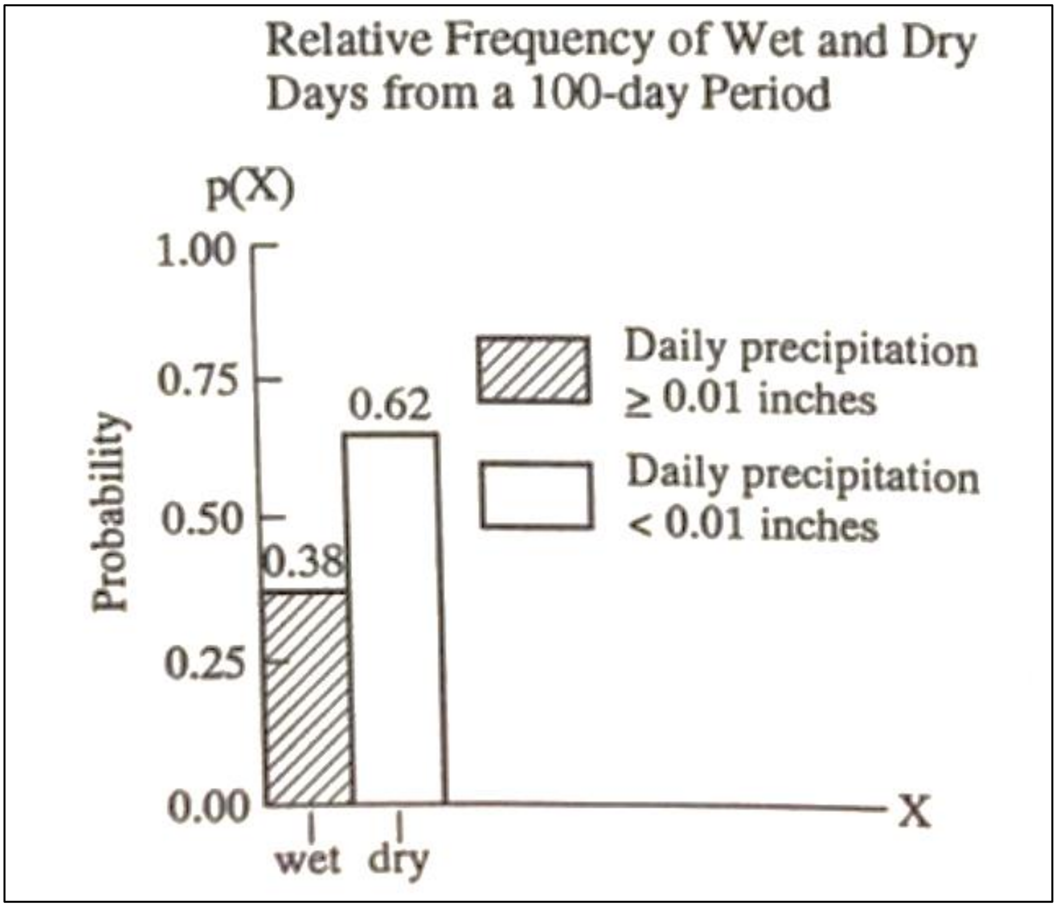
Thus, a 38 percent chance exists that a day will have measurable precipitation.

b) The probability of a dry day occurring (P,) is:

$$\text{Probability} = \frac{\text{The number of dry days}}{\text{The total days}}$$

$$P = \frac{62}{100} = 0.62$$

Thus, a 62 percent chance exists that a day will not have measurable precipitation.



Several rules guide the use of probability.

The maximum probability for any outcome is 1.0, which indicates total certainty or perfect likelihood of a particular occurrence. The lowest probability for an outcome is 0.0, which suggests no chance of this occurrence. Most outcomes have probabilities falling between the maximum (1.0) and minimum (0.0) values. Because each event actually takes place with one outcome occurring, the sum of the probabilities for all outcomes must equal 1.0.

In the previous example of precipitation, the probability of a wet day was 0.38 and the probability of a dry day was 0.62. This simple example has only two outcomes, and the probabilities clearly sum to 1.0. Many other rules of probability exist for more complex applications.