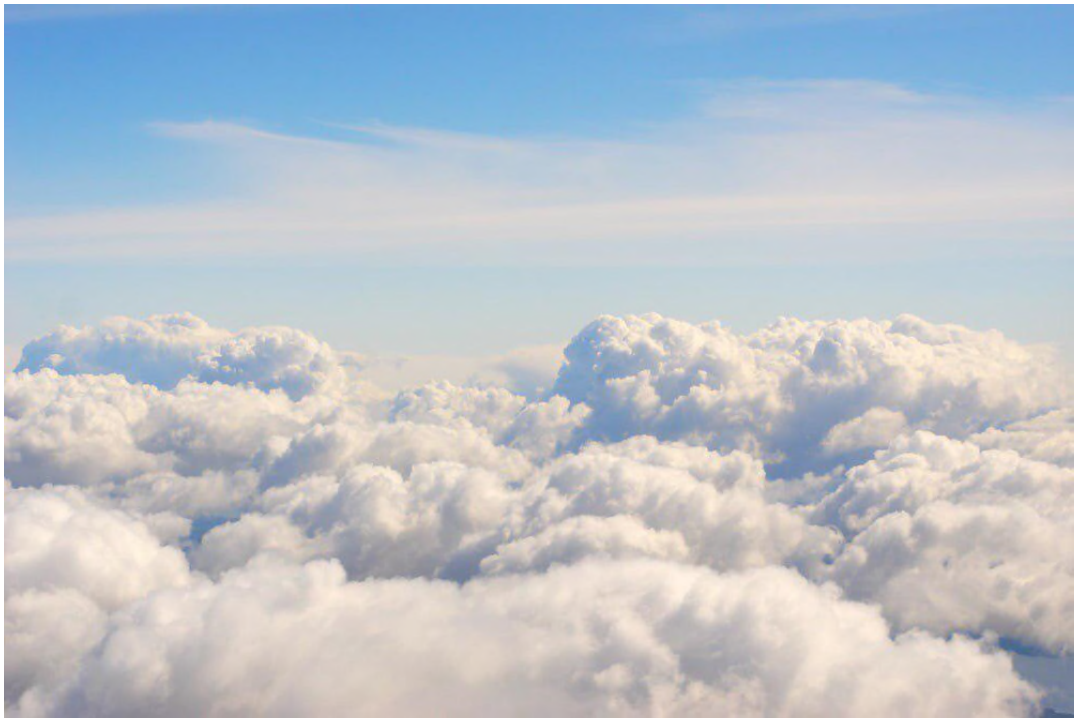
Cloud Physics



Lecture notes for
4th Year Class

by

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Chapter One

Review of Atmospheric Thermodynamics

(Part I)

Humidity Variables

This section contains a review of the humidity variables used by meteorologists.

- **Vapor pressure, e**: The partial pressure of the water vapor in the atmosphere.

- **Saturation vapor pressure, e_s**: The vapor pressure at which liquid water and vapor would be in equilibrium.

  - Saturation vapor pressure is found from the *Clausius-Clapeyron* Equation,

$$e_s = e_o \exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_o} - \frac{1}{T}\right)\right] \tag{1.1}$$

    where $T_o = 273.15$K, $e_o = 611$ Pa, L_v is the *latent heat* of vaporization, and R_v is the specific gas constant for water vapor.

  - The Clausius-Clapeyron equation gives the saturation vapor pressure over a flat surface of pure liquid water.

  - If the liquid contains impurities, or the interface between the liquid and vapor is curved, corrections for the *solute effect* and *curvature effect* must be applied.

- **Relative Humidity, *RH***: Relative humidity is defined as the ratio of vapor pressure to saturation vapor pressure, and is expressed as a percent.

$$RH = \frac{e}{e_s} \times 100\% \tag{1.2}$$

  - Relative humidity is always defined in terms of a flat surface of pure liquid water.

  - The World Meteorological Organization (WMO) uses an alternate definition of relative humidity, which is the ratio of mixing ratio to saturation mixing ratio,

$$RH = \frac{r}{r_s} \times 100\% \tag{1.3}$$

  - The two definitions (1.2) and (1.3) are not identical, but are very close and are usually considered to be interchangeable.

- **Saturation Ratio, *S***: The relative humidity expressed as a simple ratio rather than as a percent. For example, if $RH = 45\%$ then $S = 0.45$.

- **Absolute humidity, ρ_v**: The mass of water vapor per volume of air. The relation between vapor pressure and absolute humidity is given by the ideal gas law for pure water vapor, (1.11).

- **Mixing ratio, *r***: The mass of water vapor per mass of dry air. Mixing ratio is related to vapor pressure by

$$r = \frac{R_d}{R_v}\frac{e}{p-e} = \varepsilon\frac{e}{p-e} \tag{1.4}$$

where R_d and R_v are the specific gas constants for dry air and water vapor respectively, and p is the total air pressure.

- **Specific humidity, *q***: The mass of water vapor per total mass of air.
  - Mixing ratio and specific humidity are related via

$$q = \frac{r}{1+r} \tag{1.5}$$

  - Mixing ratio and specific humidity are very close in value, and are often used interchangeably.

- **Dewpoint, T_d**: The temperature to which air must be cooled at *constant pressure* and *specific humidity* in order to reach 100% relative humidity.

  - When the temperature is equal to the dewpoint temperature, the vapor pressure is equal to the saturation vapor pressure. Thus, if dewpoint temperature is used in the Clausius-Clapeyron equation the result is the vapor pressure,

$$e = e_o \exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_o} - \frac{1}{T_d}\right)\right]$$

Ideal Gas Law

- The ideal gas law is

$$pV = nRT \tag{1.6}$$

where R is the *universal gas constant*, 8.3145 J $\mathrm{mol}^{-1}\,\mathrm{K}^{-1}$.

- Equation (1.6) is not the most convenient form for meteorology. Instead we use the form

$$p = \rho R' T \tag{1.7}$$

  where R' is the specific gas constant, a unique constant for each individual gas species.

  - The specific gas constant is defined as

$$R' = R / M \tag{1.8}$$

    where M is the molar mass of the gas.

- The ideal gas law for dry air is

$$p = \rho R_d T \tag{1.9}$$

  where R_d is the specific gas constant for dry air, 287.1 J $\mathrm{kg}^{-1}\,\mathrm{K}^{-1}$.

- The ideal gas law can also be written in term of *specific volume*, $\alpha = 1/\rho$,

$$p\alpha = R_d T \tag{1.10}$$

- For pure water vapor the ideal gas law is:

$$e = \rho_v R_v T \tag{1.11}$$

  where R_v is the specific gas constant for water vapor, 461.5 J $\mathrm{kg}^{-1}\,\mathrm{K}^{-1}$, and ρ_v is the *absolute humidity*.

- For moist air, which is a mixture of dry air and water vapor, the ideal gas law is:

$$p = \rho R_d T\,(1 - q + q/\varepsilon) \tag{1.12}$$

  where q is the *specific humidity* and $\varepsilon = R_d/R_v$.

- By defining the *virtual temperature* as:

$$T_v = T\,(1 - q + q/\varepsilon) \tag{1.13}$$

  we can write (1.12) as

$$p = \rho R_d T_v \tag{1.14}$$

  - Virtual temperature is simply a definition, allowing us to use the dry-air ideal gas law for moist air by simply using the virtual temperature in place of the actual temperature.

  - Virtual temperature is not a measurable temperature. It must be calculated.

  - Substituting for the value of ε allows equation (1.13) to be written as:

$$T_v = T\,(1 + 0.61q) \tag{1.15}$$

The First Law of Thermodynamics

- The First Law of Thermodynamics states that there are two ways to change the internal energy, U of a system: 1) Add or remove heat, Q; 2) Via work, W, done on or by the system.

- Mathematically it is stated as

$$dU = dQ + dW \tag{1.16}$$

- The variables U, W, and Q are *extensive* variables, meaning that they depend on the mass of the system.

- Dividing an extensive variable by mass results in an *intensive* variable that does not depend on mass. We usually (but not always) add the word specific to such variables. Dividing (1.16) by mass results in

$$du = dq + dw \tag{1.17}$$

  where u is *internal energy per unit mass*, also called *specific internal energy*; q is *heat per unit mass*

- For *quasistatic* processes the work can be expressed as $dw = -pd\alpha$, and the First Law becomes

$$du = dq - pd\alpha \tag{1.18}$$

  - Note that for *isochoric* processes that the change in internal energy is synonymous with heat.

- Using the product rule for differentiation, $d(p\alpha) = pd\alpha + \alpha dp$, and (1.18) can be rearranged as

$$dh = dq + \alpha dp \tag{1.19}$$

  where $h = u + p\alpha$, and is the *specific enthalpy*.

  - Note that for *isobaric* processes that the change in enthalpy is synonymous with heat.

- For ideal gasses the specific internal energy is a function of temperature only, and can be expressed as $du = c_v dT$, where c_v is the *specific heat at constant volume*.

- Also, for ideal gasses the specific enthalpy is a function of temperature only, and can be expressed as $dh = c_p dT$, where c_p is the *specific heat at constant pressure*.

- Thus, for ideal gasses undergoing quasistatic processes the First Law of Thermodynamics can be written as either

$$c_v dT = dq - pd\alpha \tag{1.20}$$

or

$$c_p dT = dq + \alpha dp \tag{1.21}$$

- In meteorology the First Law of Thermodynamics is also called the *thermodynamic energy equation*, and is one of the seven governing equations for the atmosphere.

Potential Temperature

- ***Potential temperature, θ***, is defined as the temperature an air parcel would have if it were moved adiabatically to a reference pressure of p_o = 1000 mb.
  - The equation for potential temperature is derived by starting with (1.21), setting $dq = 0$ and substituting for α from the ideal gas law. The result is:

$$\frac{dT}{T} = \frac{R_d}{c_p}\frac{dp}{p} \tag{1.22}$$

Integrating (1.22) between temperatures T and θ, and between pressures p and p_o,

$$\int_T^{\theta}\frac{dT}{T} = \int_p^{p_o}\frac{R_d}{c_p}\frac{dp}{p}$$

yields the *Poisson relation* $\theta = T\left(\frac{p_o}{p}\right)^{R_d/c_p}$ (1.23)

  - Potential temperature is conserved in adiabatic motion. This means that the dry adiabatic on a Skew-T diagram are also lines of constant potential temperature.

- ***Equivalent potential temperature, θ_e***, is defined as the potential temperature the air parcel would have if it were lifted pseudo adiabatically until all of the water vapor were condensed, and the latent heat added to the parcel.
  - Equivalent potential temperature is always greater than or equal to the potential temperature.
  - Equivalent potential temperature is conserved in reversible moist-adiabatic motion.

Dry-adiabatic Lapse Rate

- Adiabatic processes are those in which no heat is transferred (dq = 0), and are of fundamental importance in atmospheric processes.
- As an air parcel rises adiabatically expands so that its pressure equilibrates to that of the surrounding environment.

- The parcel does work as it expands, and therefore loses internal energy and its temperature drops.

- The decrease in temperature with height can be found by starting with the first law of thermodynamics. For an air parcel undergoing an adiabatic process the first law of thermodynamics becomes

$$c_p dT' = \alpha' dp' \tag{1.24}$$

where the primes indicate properties of the air parcel.

  - Dividing (1.24) by dz results in

$$c_p \frac{dT'}{dz} = \alpha \frac{dp'}{dz} \tag{1.25}$$

  - The pressure of the air parcel, p' is assumed to always be equal to the pressure of the its environment, p.

  - If the environment is assumed to be in *hydrostatic balance*, then

$$\frac{dp'}{dz} = \frac{dp}{dz} = -\frac{g}{\alpha}$$

and (1.25) can be rearranged to

$$\frac{dT'}{dz} = -\frac{\alpha}{\alpha'}\frac{g}{c_p} \tag{1.26}$$

- Equation (1.26) describes how the temperature change with altitude of an unsaturated air parcel lifted adiabatically. The ratio $\frac{\alpha}{\alpha'} \approx 1$, and is therefore ignored.

- Since *lapse rate* is defined as $-\frac{dT}{dz}$, *dry-adiabatic lapse rate* as

$$\Gamma_d = -\frac{dT}{dz} = \frac{g}{c_p} \tag{1.27}$$

which has a numerical value of $\Gamma_d = 9.8^\circ\text{C km}^{-1}$.

  - The specific heat actually depends in a very small way on the humidity content of the air. This correction is small enough that it is usually ignored and so c_p in (1.27) is that for dry air.

- Even though Γ_d is called the dry-adiabatic lapse rate, it is appropriate for a moist air parcel as long as the parcel remains unsaturated.