Wind Energy 6

6- Methods Estimating the Weibull Parameters:

In this lecture, we present some methods for estimating Weibull parameters, namely, shape parameter (c) and scale parameter (k). The Weibull distribution is an important distribution especially for reliability and maintainability analysis. Weibull parameters regulate the wind speed distribution for optimum performance of a wind energy conversion system. It is, therefore very essential to accurately estimate the parameters for any candidate site to install wind energy conversion systems. Various methods have been proposed to estimate the parameters, and the suitability of each method varies with sample data distribution, which in turn, varies from location to another. The presented Methods that are used to estimate Weibull parameters (c and k) can be classified into **two categories: graphical and analytical methods**.

6-1 Graphical Methods

Usually, the graphical methods are used because of their simplicity and speed. However, they involve a great probability of error. There are two main graphical methods.

1-Weibull Probability Plotting

2-Hazard Plotting Technique

The hazard plotting technique is an estimation procedure for the Weibull parameters. This is done by plotting cumulative hazard function H(x) against failure times on a hazard paper or a simple log-log paper.

6-2 Analytical Methods

to the high probability of error in using graphical methods, it is preferring to use the analytical methods. This is motivated by the availability of high-speed computers. In the following, we discuss some of the analytical methods used in estimating Weibull parameters. These analytical methods are:

1- Maximum Likelihood Method (MLE).
2- Least Square Method (LSM).
3- Standard Deviation Method (SDM).
4- Energy Pattern Factor Method (EPFM).
5- Moment Method (MM).
6- Power Density Method (PDM).

In this lecture, the first three methods were depended to estimate Weibull parameters (c and k).

6.2-1 Maximum Likelihood Method (MLM)

Maximum likelihood method, with many required features is the most widely used technique among other parameters estimation techniques. The (MLM) has many large sample properties that make it attractive for use. It is asymptotically consistent, which means that as the sample size gets larger, the estimate converges to the true values. Let $\nu_1, \nu_2, \nu_3 \ldots\ldots \nu_n$ be a random sample size (n) drawn from a PDF $f(\nu, \theta)$ where (θ) is an unknown parameter, the likelihood function of this random sample is the joint density of (n) random variables and is a function of the unknown parameter. Thus, equation (6.1) is the likelihood function.

$$L = \prod_{i=1}^{n} f(v_i, \theta) \text{ ------------------------- } 6-1$$

The Maximum Likelihood Estimator (MLE) of (θ), say $(\overline{\theta})$, is the value of (θ) that maximizes (L), or equivalently, the logarithm of (L). Often, but not always, the MLE of(θ), is a solution of:

$$\frac{d\log L}{d\theta} = 0 \qquad 6-2$$

Now, we apply the (MLE) to estimate the Weibull parameters, namely the shape and scale parameters (*c* & *k*). Consider the Weibull probability density function (PDF) given in the equation, then the likelihood function will be:

$$L(v_1, v_2, \dots, v_n, k, c) = \prod_{i=1}^{n} \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} exp\left(-\left(\frac{v}{c}\right)^{k}\right) \text{ ----- } 6-3$$

On taking the logarithms of equ. (6-3) differentiating with respect to (*k*) and (*c*) in turn and equating to zero, we obtain the following estimating equations:

$$\frac{\partial \ln L}{\partial k} = \frac{n}{k} + \sum_{i=1}^{n} \ln v_i - \frac{1}{c}\sum_{i=1}^{n} v_i^k \ln v_i = 0 \text{ ---------------- } 6-4$$

$$\frac{\partial \ln L}{\partial c} = \frac{-n}{c} + \frac{1}{c^2}\sum_{i=1}^{n} v_i^k = 0 \text{ --------------------------------- } 6-5$$

In eliminating (*c*) between equations (6-4) and (6-5) and simplifying these equations, one can get:

$$\frac{\sum_{i=1}^{n} v_i^k \, ln\, v_i}{\sum_{i=1}^{n} v_i^k} - \frac{1}{k} - \frac{1}{n}\sum_{i=1}^{n} ln\, v_i = 0 \text{ ---------------------- } 6-6$$

After rearranging the equation (6-6), it is possible to estimate the shape parameter (*k*) as follows:

$$k = \left(\frac{\sum_{i=1}^{n} v_i^k \ln(v_i)}{\sum_{i=1}^{n} v_i^k} - \frac{\sum_{i=1}^{n} \ln(v_i)}{n}\right)^{-1} \text{-----------------} 6-7$$

Because (k) appears on both sides of the equation, the equation (6-7) must be iteratively solved. To find a convergent value for (k), several iterations are required.
Once (k) is determined, (c) can be estimated using equation (6-8) as follows:

$$c = \left(\frac{\sum_{i=1}^{n} v_i^k}{n}\right)^{1/k} \text{---------------------------------------} 6-8$$

Here, (v_i) is the wind speed in time step (i), and (n) is the number of nonzero wind speed data points.

6.2-2 Least Square Method (LSM)

The second estimating method is known as the least square method. It is so commonly applied in engineering and mathematics problems. This method is used to estimate Weibull parameters. The cumulative density function of Weibull distribution with two parameters can be written as:

$$F(v) = 1 - e^{-\left(\frac{v}{c}\right)^k} \text{-----------------------------------} 6-9$$

This function can be arranged as:

$$(1 - F(v_i))^{-1} = e^{\left(\frac{v_i}{c}\right)^k} \text{----------------------------} 6-10$$

If we took the natural logarithm of equation (10), we get:

$$-\ln(1 - F(v_i)) = \left(\frac{v_i}{c}\right)^k \text{-------------------------} 6-11$$

And then to retake the natural logarithm of equation (11), we get the following equation:

$$\ln[-\ln(1 - F(v_i))] = k \ln v_i - k \ln c \text{----------------} 6-12$$

This is in the form of an equation of a straight line:

$$y_i = ax_i + b \text{ -------------------------------------- } 6-13$$

Where (x_i) and (y_i) are variables, (a) is the slope, and (b) is the intercept of the line on the (y) axis, such that:

$$y_i = \ln\left[-\ln\left(1 - F(v_i)\right)\right]$$

$$a = k \quad ,$$

$$b = -k \ln c - -\text{-----------------------------------}6-14$$

The idea is to determine the values of (a) and (b) in equation (13) such that the straight line drawn through the (x_i, y_i) points has the best possible fit. (k) and (c) Parameters are given by:

$$k = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \text{ --------------------}6-15$$

$$c = \exp\left(\frac{k \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i}{nk}\right) \text{ ----------------------}6-16$$

6.2-3 Standard Deviation Method (SDM)

This method is useful where only the mean wind speed and standard deviation are available. In addition, it has relatively simple expressions when compared with other methods. Moreover, it is unlike most of the other methods that may require more detailed wind data to determinate the Weibull distribution shape and scale parameters. The shape and scale parameters can be estimated from the mean and standard deviation of wind data, considering the expressions of mean and standard deviation given, we get:

$$(\frac{\sigma}{v_m})^2 = \frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} - 1 \text{ ------------------------} 6-18$$

Once (v_m) and (σ) are calculated for given data, the value of (k) can be found easily by solving the above expression numerically. Acceptable approximation for (k) is given by:

$$k = \left(\frac{\sigma}{v_m}\right)^{-1.086} \text{ ----------------------------------} 6-19$$

The scale parameter (c) is given by:

$$c = \frac{v_m}{\Gamma\left(1+\frac{1}{k}\right)} \text{ ----------------------------------} 6-20$$

More accurately, (c) can be found using the following expression.

$$c = \frac{v_m k^{2.6674}}{0.184 + 0.816\, k^{2.73855}} \text{ --------------------} 6-21$$