الجامعة المستنصرية
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المرحلة الثالثة / مسائي

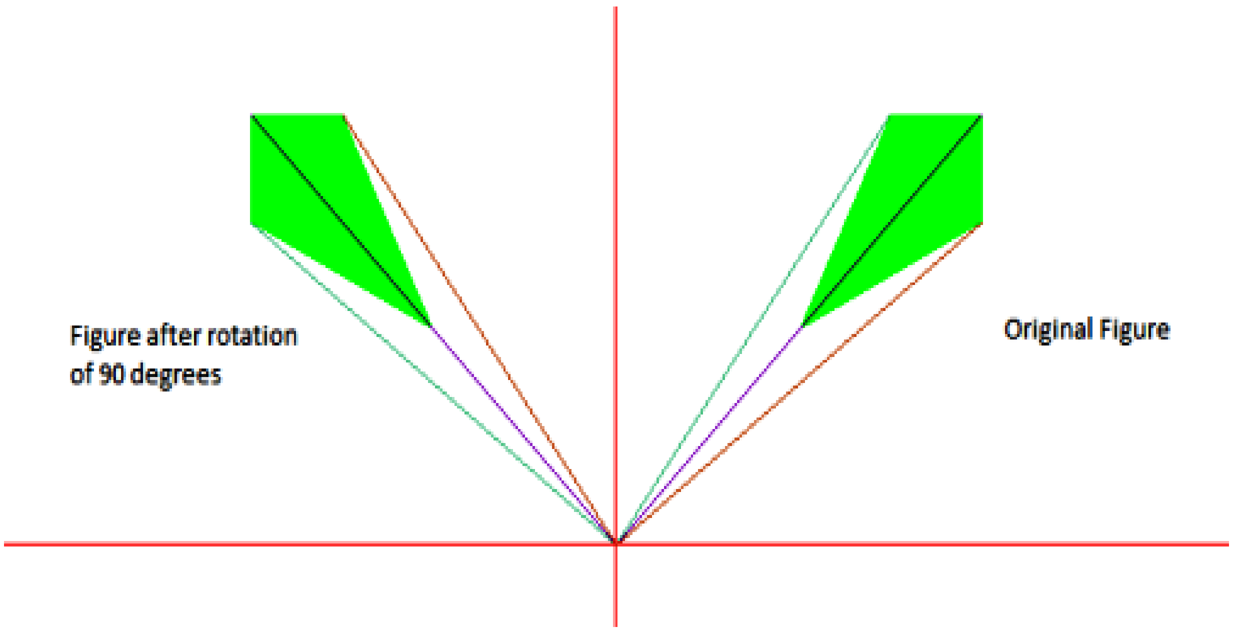
Computer Graphics

LECTURE 10

أ.م. صلاح طه علاوي

Two Dimension Transformation

Rotation



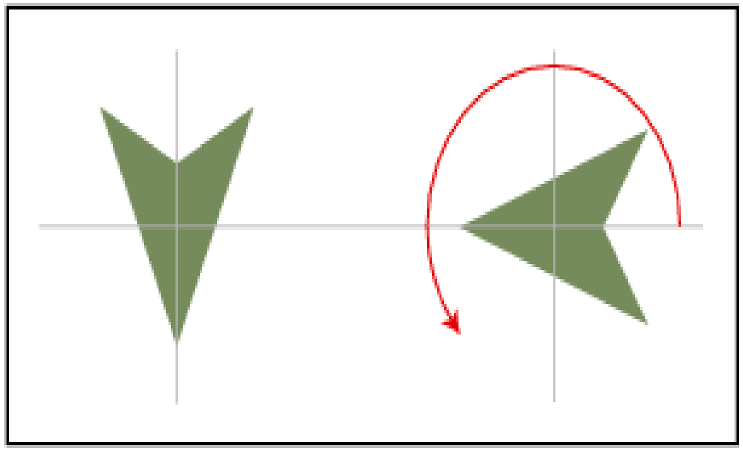
3- Rotation

Another useful transformation is the rotation of an object about specified pivot point. In rotation, the object is rotated Ó about the origin.

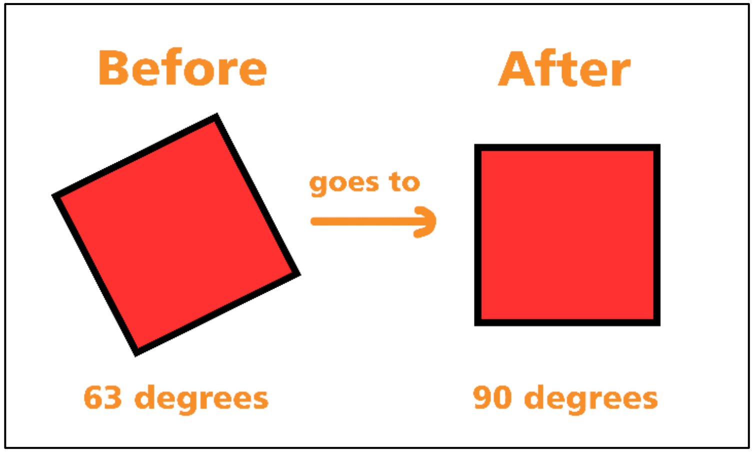
The convention is that :

1- The direction of rotation is counterclockwise if Ó is a positive angle

2- The direction of rotation is clockwise if Ó is a negative angle.



(a) counterclockwise if Ó is a positive angle



(b) clockwise if Ó is a negative angle

Rotation about the origin

The rotation matrix to rotate an object about the origin in anticlockwise direction is:

Cos θ	Sin θ	0
-Sin θ	Cos θ	0
0	0	1

Alternatively, in equation:

$$X_{new} = X * \text{Cos}\,\theta - Y * \text{Sin}\,\theta$$

$$Y_{new} = Y * \text{Cos}\,\theta + X * \text{Sin}\,\theta$$

The form of the rotation matrix to rotate an object about the origin in anticlockwise direction :

Cos θ	Sin θ	0
-Sin θ	Cos θ	0
0	0	1

When Ǿ =90

Sin (90) = 1

Cos (90) = 0

0	1	0
-1	0	0
0	0	1

When Ǿ =180

Sin (180) = 0

Cos (180) = -1

-1	0	0
0	-1	0
0	0	1

When Ǿ =270

Sin (270) = -1

Cos (270) = 0

0	-1	0
1	0	0
0	0	1

When Ǿ =360

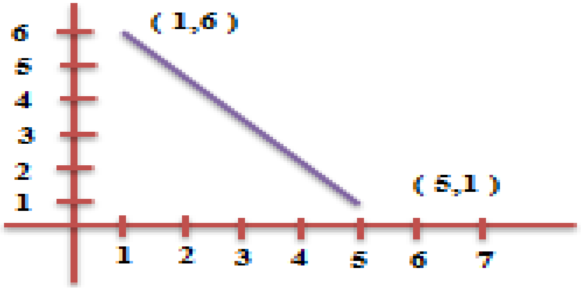
Sin (360) = 0

Cos (360) = 1

1	0	0
0	1	0
0	0	1

Example 1:

Rotate the line P1(1,6) and P2(5,1) anticlockwise 90 degree.



Original line

The solve:

$$X_{new} = X * \text{Cos}\,\theta - Y * \text{Sin}\,\theta$$

$$Y_{new} = Y * \text{Cos}\,\theta + X * \text{Sin}\,\theta$$

First point

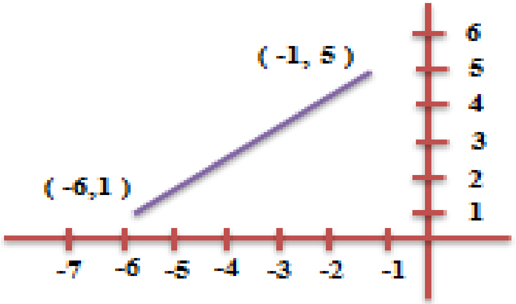
X_{new} =1*0 – 6* 1 = -6

y_{new} = 6*0 + 1*1 = 1

Second point

X_{new} =5*0 – 1* 1 = -1

y_{new} = 1*0 + 5*1 = 5

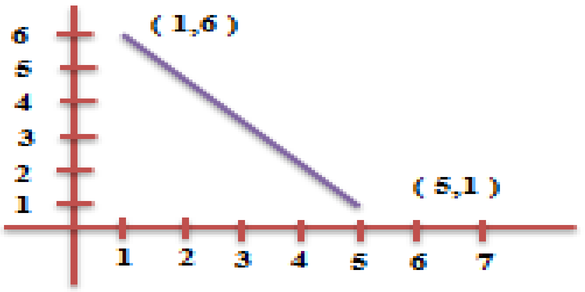


Line after rotation

anticlockwise if Ó is a positive angle

Example 1:

Rotate the line P1(1,6) and P2(5,1) anticlockwise 90 degree.



Original line

The solve:

X1 new	Y1 new	1
X2 new	Y2 new	1

=

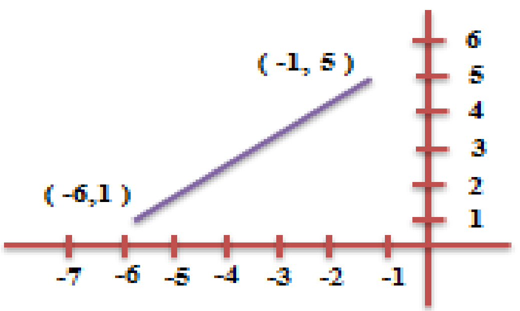
1	6	1
5	1	1

*

0	1	0
-1	0	0
0	0	1

=

-6	1	1
-1	5	1



Line after rotation

anticlockwise if Ø is a positive angle

Note:

Rotation in clockwise direction:

In order to rotate in clockwise direction we use a negative angle, and because:

Cos (-Ǿ) = Cos Ǿ

Sin (-Ǿ) = - Sin Ǿ

Therefore, The form of the rotation matrix to rotate an object about the origin in clockwise direction :

Cos Ǿ	-Sin Ǿ	0
Sin Ǿ	Cos Ǿ	0
0	0	1

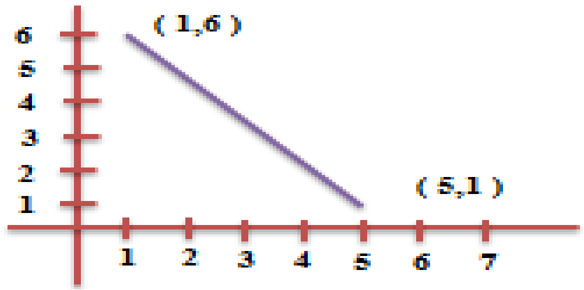
Alternatively, in equation:

Xnew=X * Cos Ǿ + Y * Sin Ǿ

Ynew= Y * Cos Ǿ - X * Sin Ǿ

Example 2:

Rotate the line P1(1,6) and P2(5,1) in clockwise (-90) degree.



Original line

The solve:

X1 new	Y1 new	1
X2 new	Y2 new	1

=

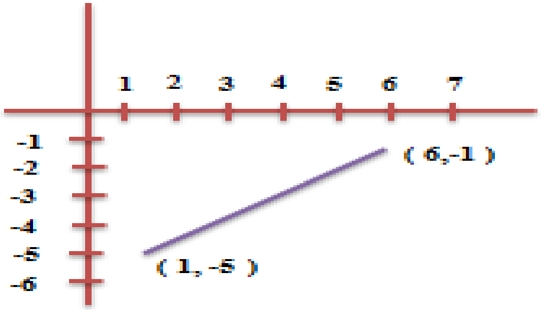
1	6	1
5	1	1

*

0	-1	0
1	0	0
0	0	1

=

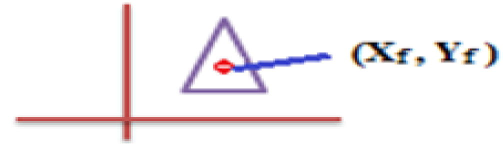
6	-1	1
1	-5	1



Line after rotation

clockwise if Ø is a negative angle

Rotate about a specific point (XP, YP)



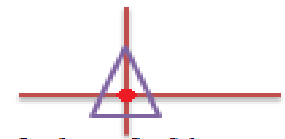
original position of object and pivot point

We need three steps:

1- Translate the points (and the object) so that the point (XP,YP) lies on the origin

XP1= X - XP

YP1= Y – YP

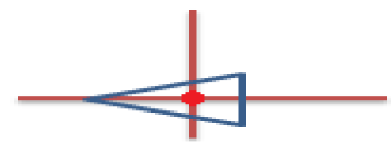


translation of object so that pivot point (Xf , Yf) is at origin

2- Rotate the translated point (and the translated object) by Ǿ degree about the origin to obtain the new point (XP2,YP2)

XP2=XP1 * Cos Ǿ - YP1 * Sin Ǿ

YP2=YP1 * Cos Ǿ + XP1 * Sin Ǿ

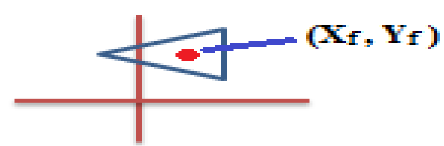


rotation about origin

3- Back translation

XP3= XP2 + XP

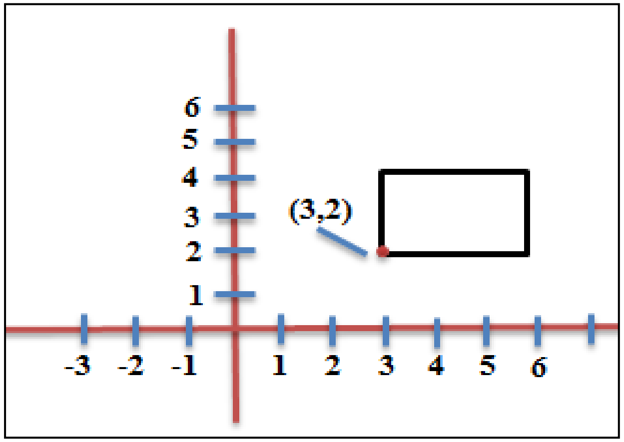
YP3= YP2 + YP



translation of object so that pivot point is returned to position (Xf , Yf)

Example 3:

Rotate the rectangle (3,2),(6,2),(3,4),(6,4) counterclockwise with Ǿ =90 around the point (3,2).



The solve:

1- Translation

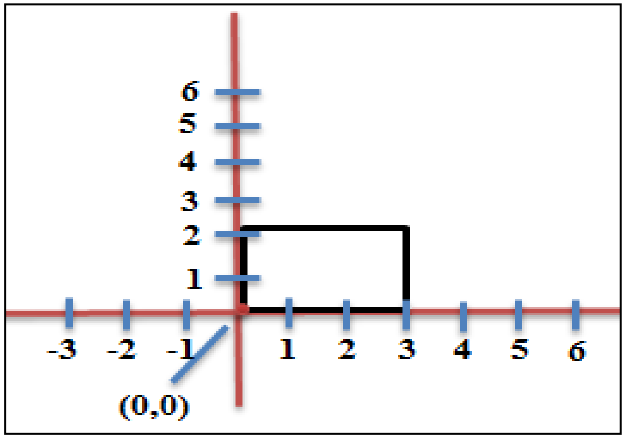
XP= 3 , YP= 2

XP1= X - XP

YP1= Y – YP

Point (3,2)	Point (6,2)
$X1_{new} = 3 - 3 = 0$ $Y1_{new} = 2 - 2 = 0$	$X2_{new} = 6 - 3 = 3$ $Y2_{new} = 2 - 2 = 0$

Point (3,4)	Point (6,4)
$X3_{new} = 3 - 3 = 0$ $Y3_{new} = 4 - 2 = 2$	$X4_{new} = 6 - 3 = 3$ $Y4_{new} = 4 - 2 = 2$



2- Rotation

When Ó =90

Sin (90) = 1

Cos (90) = 0

XP2=XP1 * Cos Ó - YP1 * Sin Ó

YP2=YP1 * Cos Ó + XP1 * Sin Ó

0	1	0
-1	0	0
0	0	1

Point (0,0)

$X1_{new}$= 0 * (0) – 0 * (1) = 0

$Y1_{new}$= 0 * (0) + 0 * (1) = 0

Point (0,2)

$X3_{new}$= 0 * (0) – 2 * (1) = -2

$Y3_{new}$= 2 * (0) + 0 * (1) = 0

Point (3,0)

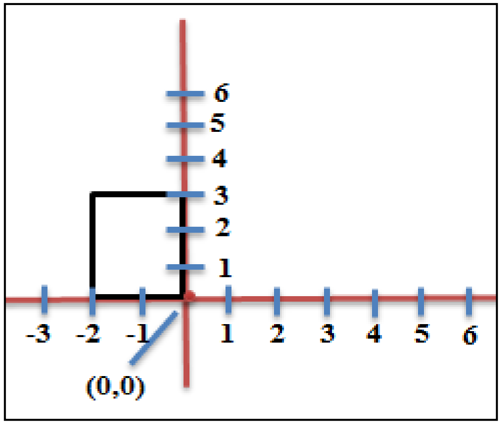
$X2_{new}$= 3 * (0) – 0 * (1) = 0

$Y2_{new}$= 0 * (0) + 3 * (1) = 3

Point (3,2)

$X4_{new}$= 3 * (0) – 2 * (1) = -2

$Y4_{new}$= 2 * (0) + 3 * (1) = 3



3- Back Translation

$XP3 = XP2 + XP$

$YP3 = YP2 + YP$

Point (0,0)

$X1_{new} = 0 + 3 = 3$

$Y1_{new} = 0 + 2 = 2$

Point (-2,0)

$X3_{new} = -2 + 3 = 1$

$Y3_{new} = 0 + 2 = 2$

Point (0,3)

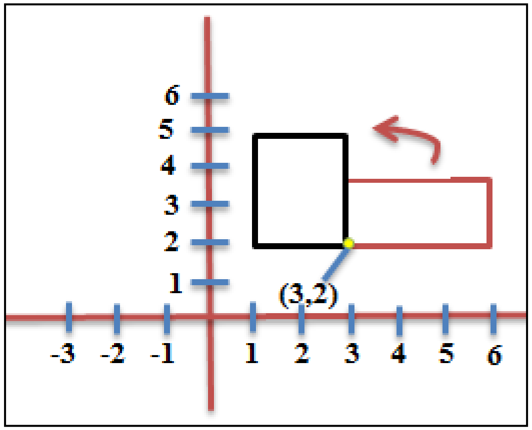
$X2_{new} = 0 + 3 = 3$

$Y2_{new} = 3 + 2 = 5$

Point (-2,3)

$X4_{new} = -2 + 3 = 1$

$Y4_{new} = 3 + 2 = 5$



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