Chapter (2)

Energy Band-gap Theory

Scientific Terms and Formulas to Remember

Generally, the conduction in the crystal (such as semiconductors materials) depends on the number of charge carriers. The number of charge carrier that contributes to the conduction process is the function of the number of available energy or quantum states. The density of these allowed energy state is called the density of state function. This helps to calculate the hole and electron concentration.

Important Scientific Terms :

$n_i^2 = n_o \ P_o$: Intrinsic carrier concentration

E_{Fi} : intrinsic Fermi energy

N_d or N_D : It is the donor concentration, for an n-type material at equilibrium:

$$n \approx N_d \ cm^{-3}$$

where n_o is the number of conducting electrons (electron concentration)

N_a or N_A: It is the acceptor concentration, for a p-type material at equilibrium:

$$P_o \approx N_a \quad cm^{-3}$$

where P_o is the hole concentration.

N_c : It is called the effective density of energy states (or quantum states) in the conduction band.

N_v : It is called the effective density of energy states (or quantum states) in the Valence band.

Important Formulas :

1. Band-gap Energy $(E_g) = (E_c - E_v)$

$$n_0 p_0 = n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

2. Position of the fermi level in the n-type semiconductor in terms of N_c and n_0 is given by

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right)$$

3. Position of the fermi level in the n-type semiconductor in terms of N_d and N_c is given by

$$E_c - E_F = kT \ln\left(\frac{N_c}{N_d}\right)$$

[Since, in n-type semiconductor we have, $N_d > n_i$ and $n_0 \approx N_d$]

4. Position of the fermi level in the p-type semiconductor in terms of N_a and N_v is given by

$$E_F - E_v = kT \ln\left(\frac{N_v}{N_a}\right)$$

5. Concentration of electrons in the conduction band in terms of n_i is given by

$$n_0 = n_i \exp\left(\frac{(E_F - E_{Fi})}{kT}\right)$$

6. Concentration of holes in the valence band in terms of n_i is given by

$$p_0 = n_i \exp\left(\frac{-(E_F - E_{Fi})}{kT}\right)$$

7. Position of the fermi level in the n-type semiconductor in terms of n_o and n_i is given by

$$E_F - E_{Fi} = kT \ln\left(\frac{n_o}{n_i}\right)$$

8. Position of the fermi level in the p-type semiconductor in terms of p_o and n_i is given by

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

ملاحظة/ عند قرائتك للمصادر العلمية المختلفة المتعلقة بهذا الموضوع ممكن ان يكون هناك اختلاف بسيط في استخدام الرموز العلمية او قيمة الثوابت من قبل المؤلفين لذلك انا تعمدت ان اضع رمزين بعض الاحيان مثلا electron concentration (n or n_0) والتي تدل على نفس المعنى العلمي للمطلح المستخدم وهذا من اجل ان لا يحصل عند الطالب التباس اثناء القراءة .

These are examples presenting the relationship between doping, carrier concentrations and the Fermi energy level at equilibrium condition

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Required Constants: For silicon (Si) at T=300 K, $n_i = 1.5 \times 10^{10}$ cm^{-3}. The Boltzmann constant $k_B = k = 8.61 \times 10^{-5}$ eV/K. Silicon band-gap energy $E_g = 1.12$ eV.

Q1/ The electron concentration in Silicon (Si) at T = 300 K is $n_o = 5 \times 10^4$ cm $^{-3}$? (a) Determine hole concentration (P_o). Is this n or p-type material? (b) Determine the position of the Fermi energy level (E_F) with respect to the intrinsic Fermi energy level (E_{Fi}). If n_i of (Si) is about 1.5×10^{10} cm^{-3}

Answer: $n_0 = 5 \times 10^4$ at T = 300 K

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^4} = 4.5 \times 10^{15}\ cm^{-3}$$

As $p_0 > n_0$ the material is p-type semiconductor.

We know that

$$p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right] \Rightarrow \quad E_{Fi} - E_F = kT \ln\left[\frac{(p_0)}{n_i}\right]$$

$$E_{Fi} - E_F = 0.0259 \ln\left[\frac{4.5 \times 10^{15}}{1.5 \times 10^{10}}\right] = 0.3266\ eV$$

Q2/ The value of p_0 in Silicon at T = 300 K is 10^{15} cm^{-3}. Determine $E_C - E_F$ and n_0.

Answer $p_0 = 10^{15}\ cm^{-3}$ at T = 300 K

$$p_0 = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$$\Rightarrow \quad E_F - E_V = kT \ln\left[\frac{N_V}{p_0}\right] = 0.0259 \ln\left[\frac{1.04 \times 10^{19}}{10^{15}}\right] = 0.24\ eV$$

$$\Rightarrow \quad E_C - E_F = 1.12 - 0.24 = 0.88\ eV$$

n_o = **?** (H. W.)

Q3 (A)/ Consider a silicon crystal at room temperature (300 K) doped with arsenic (As) atoms so that $N_D = 6 \times 10^{16}\ cm^{-3}$. Find (1) the equilibrium electron concentration (n or n_o), (2) hole concentration p_o, and (3) Fermi energy level (E_f or E_F) with respect to the intrinsic Fermi level (E_{fi} or E_i) and conduction band edge E_C.

Required Constants: For silicon (Si) at T=300 K, $n_i = 1.45 \times 10^{10}\ cm^{-3}$. The Boltzmann constant $k = 8.61 \times 10^{-5}$ eV/K. Silicon band-gap energy $E_{g\,(si)} = 1.12$ eV.

Answer :

(1) This is an n-type material, as it is doped with donor atoms (N_D or N_d). Therefore:

$$(n \text{ or } n_o) = N_D = 6 \times 10^{16}\ cm^{-3} \qquad (1)$$

(2) Then we can use the Law of Mass Action to find the (hole concentration) :

$$p_0 = \frac{n_i^2}{n_0} = \frac{2.1 \times 10^{20}}{6 \times 10^{16}} = 3.5 \times 10^3 \ 1/\text{cm}^3 \quad (2)$$

(3) To find the Fermi level with respect to the intrinsic Fermi level, we use the expression that links electron concentration to E_i and n_i:

$$n_0 = n_i \exp(\frac{E_F - E_i}{kT}) \Rightarrow (E_F - E_i) = kT \ln(\frac{n_0}{n_i}) \quad (3)$$

At room temperature

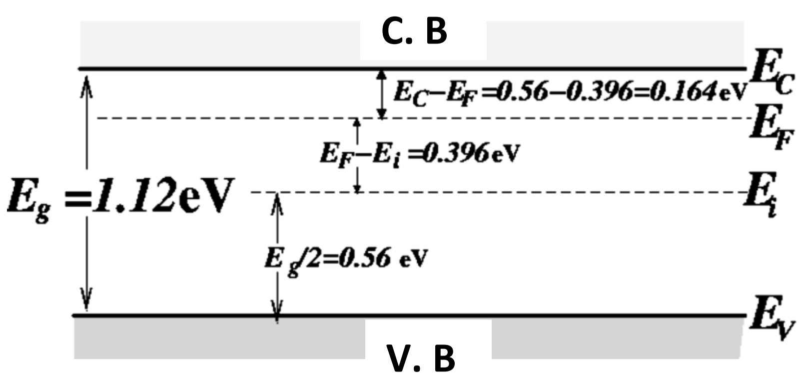
$$kT = 8.61 \times 10^{-5} \times 300 \approx 0.026 \text{ eV} \quad (4)$$

We will be using this quantity often.

Then the separation of the Fermi level and intrinsic Fermi level is, from Eqn. 3:

$$\begin{aligned}(E_F - E_i) &= 0.026 \ln(\frac{6 \times 10^{16}}{1.45 \times 10^{10}}) = 0.026 \times 15.24 = 0.396 \text{ eV} \\ \Rightarrow E_F &= E_i + 0.396 \text{ eV} \quad (5)\end{aligned}$$

From drawing the band-gap energy (E_g) diagram, we can then place the Fermi energy level (E_F) in the correct place with respect to the intrinsic Fermi energy level (E_i or E_{fi}) (middle of band-gap energy (E_g)) and also find its separation from the conduction band edge E_C.



H. W (Homework)

Q3 (B) (H. W.) / Consider a silicon (Si) crystal at 300 K, with the Fermi energy level 0.15 eV above the valence band. What type is the material? Calculate the electron and hole concentrations?