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Theorems-11

Let a_0, a_1, a_2 be given Continous functions on some intervals I and let $a_0(t) \neq 0$ for all t then the solution $\emptyset_1$ and $\emptyset_2$ of

$L[y] = a_0(t) y'' + a_1(t) y' + a_2(t) y = 0$ on I.

are L-indep iff $W(\emptyset_1, \emptyset_2)(t) \neq 0$, $\forall t \in R$.

proof:-

Suppose first that the solution $\emptyset_1$ and $\emptyset_2$ of $L[y] = 0$ s.t $W(\emptyset_1, \emptyset_2)(t) \neq 0$ for all $t \in I$.

By def. of L-indep.

there exist aconstant b_1, b_2 s.t.

not both Zero s.t.

$b_1 \emptyset_1(t) + b_2 \emptyset_2(t) = 0 \quad \forall t \in I.$

$b_1 \emptyset_1'(t) + b_2 \emptyset_2'(t) = 0 \quad \forall t \in I.$

for each fixed $t \in I$.

the determinent of their Coffcients is

$W(\emptyset_1, \emptyset_2)(t)$ but $W(\emptyset_1, \emptyset_2)(t) \neq 0 \quad \forall t \in I$.

$b_1 = b_2 = 0$

$\emptyset_1, \emptyset_2$ are L-indep.

Now Secondly:-

Assume the Solution $\emptyset_1$ and $\emptyset_2$ are L-indep. on I.

and Assume that there's $\hat{t}$ s.t.

$W(\emptyset_1, \emptyset_2)(\hat{t}) = 0$

$b_1 \emptyset_1(\hat{t}) + b_2 \emptyset_2(\hat{t}) = 0$

$b_1 \emptyset_1'(\hat{t}) + b_2 \emptyset_2'(\hat{t}) = 0$

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- So the system have at least one solution b_1, b_2 where not both are Zero.

Define the functions:-

$\psi(t) = b_1 \phi_1(t) + b_2 \phi_2(t)$.

first ψ is asolution of $L[y] = 0$.

$\psi(t^n) = \psi'(t^n) = 0$

$\psi(t) = 0 \quad \forall t \in I$.

So...

$b_1 \phi_1(t) + b_2 \phi_2(t) = 0 \quad \forall t \in I$.

$\Rightarrow w(\phi_1, \phi_2)(t) \neq 0$.

Q..

Check:-

if Q_1 and Q_2 are not solution of differtial equation then Theorem 11 can be applied on any two functions p(t), q(t) : conts func.

Corollary :- 3 :-
Two solutions φ_1, φ_2 of $y'' + p(t)y' + q(t)y = 0$.
on I are L-indep on I iff

$$W[\varphi_1(t), \varphi_2(t)] = \det \begin{bmatrix} \varphi_1(t) & \varphi_2(t) \\ \varphi_1'(t) & \varphi_2'(t) \end{bmatrix}.$$

is different from Zero on I.

Corollary : (4)
The set of solutions $\varphi_1, \ldots, \varphi_n$ of
$y^{(n)} + P_1(t) y^{(n-1)} + P_2(t) y^{(n-2)} + \cdots + P_n(t) y = 0$.
on I. are L-indep on I iff.

$$W[\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t)] = \det \begin{bmatrix} \varphi_1(t) & & \varphi_n(t) \\ \vdots & & \\ \varphi_1^{(n-1)}(t) & & \varphi_n^{(n-1)} \end{bmatrix}$$

is different from Zero on I.

Non Homog. Linear System :-

$$y' = A(t)\, y + g(t). \quad \text{---} (1)$$

Theorem :- 25 :-
If Φ is fundamental matrix of $y' = A(t)y$
on I then the function

$$\varphi(t) = \Phi(t) \int_{t_0}^{t} \Phi^{-1}(s)\, g(s)\, ds.$$

is the unique solution on I of (1) satisfying
$\varphi(t_0) = 0$.

adj (k×1)
$(-1)^{1+1}$

$(-1)^{1+2}$ / /   التاريخ   الموضوع :

Proof:-

Let $\psi(t) = \Phi(t)\, v(t)$, where v is a vector to be determined ($v(t) \neq c$ constant vector).

$\psi'(t) = \Phi'(t)\, v(t) + \Phi(t)\, v'(t)$ ..... (2)

but

$\Phi'(t) = A(t)\, \Phi(t)$, on I.

hence (2) will be

$\Phi(t)\, v'(t) = g(t)$. $\Rightarrow v'(t) = \Phi^{-1}(t) . g(t)$.

$v(t) = \int_{t_0}^{t} \Phi^{-1}(s) . g(s) . ds$, $t_0, t \in I$ ..... (3).

Now,

If we define ψ by (3).

then by differentiating and using fundamental theorem of calculus

$\psi'(t) = \Phi'(t) \int_{t_0}^{t} \Phi^{-1}(s) . g(s) . ds + \Phi(t) . \Phi^{-1}(t) . g(t)$.

$\psi'(t) = A(t) . \Phi(t) \int_{t_0}^{t} \Phi^{-1}(s) . g(s) . ds + g(t)$.

$\psi'(t) = A(t)\, \psi(t) + g(t)$ $\forall t \in I$.

also...

$\psi(t_0) = 0$.

Remarks The general solution of the non-homog. system on I. has the form $\phi_h(t) = \Phi(t) . Y$, $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$\phi(t) = \phi_h + \psi_p$

where ϕ_h is the general solution of homog. system satisfying the initial condition $\phi_h(t_0) = c$. c is any constant vector. and ψ_p is particular solution of non-homog.

Theorem:- 28:-

Let A be a constant matrix (real or complex). Suppose $v_1, v_2, \ldots, v_n$ are n-L-indep. eigen vectors corresponding respectively to eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$ then.

$$\Phi(t) = [e^{\lambda_1 t} v_1, e^{\lambda_2 t} v_2, \ldots, e^{\lambda_n t} v_n]$$

is a fundemental matrix of the linear system with constant cofficients

$$y' = Ay \quad \text{on } -\infty < t < \infty$$

In particular this case if the eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$ are distinct.

proof:

Suppose that matrix has n-Lin-indep. eigen vectors crossponding to the $\lambda_1, \lambda_2, \ldots, \lambda_n$.

So if we let

$$\phi_j = e^{\lambda_j t} v_j, \quad j = 1, 2, \ldots, n \qquad (A - \lambda I)V = 0$$

then $\phi_j(t)$ is a solution of $y' = Ay$ $\forall j = 1, \ldots, n$. $\quad AV = \lambda V$

for

$$\phi_j' = \lambda_j e^{\lambda_j t} v_j \qquad \lambda V = AV$$

$$= \lambda_j e^{\lambda_j t} v_j \qquad \lambda V - AV = 0 \qquad (A - \lambda I)V = 0$$

$$= e^{\lambda_j t} A v_j = A e^{\lambda_j t} v_j = A \phi_j(t). \quad j = 1, \ldots, n$$

Define

$$\Phi(t) = [\phi_1(t) \ldots \phi_n(t)].$$

Φ is a solution matrix

$$\det \Phi(0) = \det [v_1, \ldots, v_n] \neq 0.$$

hence Φ is fundementat matrix.