c.c1 / c1 "مناقشة"

بما ان مقدار الخطأ عند اخذ ϕ_n يحسب بواسطة

$$|\phi(t) - \phi_n(t)| \le \frac{M}{K} \frac{(Kq)^{n+1}}{(n+1)!} e^{Kq} \quad \cdots (*)$$

نأخذ مثال لحساب هذه القيمة

Ex: Let $y' = y^2 + \cos t^2$, $y(0) = 0$

find n to get the error between ϕ_n and ϕ less than 0.01, where $|t| \le \frac{1}{2}$, $|y| \le 1$.

Solution:

$$f(t,y) = y^2 + \cos t^2$$

$M = 1^2 + 1 = 2$ اي ($|f(t,y)| \le M$)

$\frac{b}{M} = \frac{1}{2} = q$ اي ($q = \frac{b}{M}$)

$\therefore b = 1$, $q = \frac{1}{2}$ then $\left|\frac{\partial f(t,y)}{\partial y}\right| = |2y| \le 2$ ... (2)

اي $|y| \le 1$

من (*)

$$|\phi(t) - \phi_n(t)| \le \frac{2}{2} \left(\frac{(2\frac{1}{2})^{n+1}}{(n+1)!}\right) e^{2(\frac{1}{2})} \quad \therefore K = 2$$

$$= \frac{e}{(n+1)!}$$

$\therefore$ error $= 0.01 \Rightarrow \frac{e}{(n+1)!} < 0.01$

$(n+1)! > 100\, e = 100(2.718) = 271.8$

$5! = 120 < 271.8 < 720 = 6!$

$\therefore 271.8 \simeq 120$

$(n+1)! > 5!$

$n+1 > 5 \rightarrow n > 4 \Rightarrow \therefore n = 5$

ϕ_5 تحقق التقريب المطلوب

Ex : Express as a single second order equation the 2x2 system and solve it,

$$x_1' = -x_1 + 3x_2 \quad \ldots (1)$$

$$x_2' = x_1 - x_2 \quad \ldots (2)$$

Solution:

from eq. (2) ; $x_1 = x_2' + x_2$

نفوض في (1)

$$(x_2' + x_2)' = -(x_2' + x_2) + 3x_2$$

$$x_2'' + x_2' = -x_2' + 2x_2$$

$$x_2'' + 2x_2' - 2x_2 = 0$$

$$m^2 + 2m - 2 = 0 \Rightarrow m = -1 \mp \sqrt{3}$$

$$\therefore x_2(t) = c_1 e^{(-1+\sqrt{3})t} + c_2 e^{(-1-\sqrt{3})t}$$

$$x_1(t) = d_1 e^{(-1+\sqrt{3})t} + d_2 e^{(-1-\sqrt{3})t}$$