(ii) (Low of Syllogism) Show that the logical proposition

$$[(p \to q) \land (q \to r)] \to (p \to q)$$

is a tautology.

Definition 1.3.8. (Logically equivalent)

Propositions r and s are logically equivalent if the truth tables of r and s are the same and denoted by $(r \equiv s)$.

Example 1.3.9. Show that

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$$(p \to q) \land (q \to p) \equiv p \leftrightarrow q.$$

Solution. Show the truth values of both propositions are identical.

| T T F T F T F T F T T F T F T F F F F T T F F F F T T F F | р | q | ~q | $p \rightarrow q$ | $\sim (p \rightarrow q)$ | p ∧ ~q |
|---|---|---|----|-------------------|--------------------------|--------|
| TFTFTTFTFFFFFFTTFF | Т | Т | F | Т | F | F |
| FTFFFFTTFFTF | Т | F | Т | F | Т | Т |
| FFT T FF | F | Т | F | Т | F | F |
| | F | F | Т | T | F | F |

Theorem 1.3.10. (Relation Between Logical Equivalent and Tautology)

 $r \equiv s$ if and only if the statement $r \leftrightarrow s$ is a tautology.

Dr. Bassam Al-Asadi and Dr. Emad Al-Zangana