

(ii) (Law of Syllogism) Show that the logical proposition

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

is a tautology.

Definition 1.3.8. (Logically equivalent)

Propositions r and s are logically equivalent if the truth tables of r and s are the same and denoted by $(r \equiv s)$.

Example 1.3.9. Show that

$$(p \rightarrow q) \wedge (q \rightarrow p) \equiv p \leftrightarrow q.$$

Solution. Show the truth values of both propositions are identical.

p	q	$\sim q$	$p \rightarrow q$	$\sim(p \rightarrow q)$	$p \wedge \sim q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

Theorem 1.3.10. (Relation Between Logical Equivalent and Tautology)

$r \equiv s$ if and only if the statement $r \leftrightarrow s$ is a tautology.