

## 1.4. Rules of Proof

### (i) Rule of Replacement.

Any term in a logical formula may be replaced by an equivalent term.

For instance, if  $q \equiv r$ , then  $p \wedge q \equiv p \wedge r$  Rep( $q:r$ ).

### (ii) Rule of Substitution.

A sentence which is obtained by substituting logical propositions for the terms of a theorem is itself a theorem.

For instance,  $(p \rightarrow q) \vee w \equiv w \vee (p \rightarrow q)$  Sub( $p: p \rightarrow q$ ), Theorem  $p \vee w \equiv w \vee p$ .

### (iii) Rule of Inference.

$\frac{p}{p \rightarrow q} \therefore q$	$\frac{\sim q}{p \rightarrow q} \therefore \sim p$
$\frac{p \rightarrow q}{q \rightarrow r} \therefore p \rightarrow r$	$\frac{p \vee q}{\sim p} \therefore q$
$\frac{p}{p} \therefore p \vee r$	$\frac{p \wedge q}{p} \therefore p$
$\frac{p}{q} \therefore p \wedge q$	$\frac{p \vee q}{\sim p \vee r} \therefore q \vee r$