Example 1.4.1. Given

(1) "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on"

(2) "If the sailing race is held, then the trophy will be awarded"

(3) "The trophy was not awarded"

Does this imply that: "It rained"?

Solution.

p: rain

q: foggy

r: the sailing race will be held

s: the lifesaving demonstration will go on

t: then the trophy will be awarded

Symbolically, the proposition is

(1) $\sim p \vee \sim q \rightarrow r \wedge s$

(2) $s \rightarrow t$

(3) $\sim t$

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p

1. $\sim t$ 3rd hypothesis
2. $s \rightarrow t$ 2nd hypothesis
3. $\sim t \rightarrow \sim s$ Contrapositive of 2
4. $\sim s$ inf (1),(3)
5. $\sim p \vee \sim q \rightarrow r \wedge s$ 1st hypothesis
6. $\sim (r \wedge s) \rightarrow \sim (\sim p \vee \sim q)$ Contrapositive of 5
7. $\sim r \vee \sim s \rightarrow (p \wedge q)$ De Morgan's law and double negation law from 5
8. $\sim r \vee \sim s$ inf (4)
9. $p \wedge q$ inf (7),(8)
10. p inf (9)

Example 1.4.2. Use the logical equivalences to show that

(i) $\sim (p \rightarrow q) \equiv p \wedge \sim q$,

(ii) $\sim (p \vee \sim (p \wedge q))$ is a contradiction,

(iii) $\sim (p \vee (\sim p \wedge q)) \equiv (\sim p \wedge \sim q)$,

(iv) $p \vee (p \wedge q) \equiv p$ (Absorption Law).

Solution.

(i) $\sim (p \rightarrow q) \equiv \sim (\sim p \vee q)$ Implication Law