$\equiv \sim(\sim p) \wedge \sim q.$ De Morgan's Law
$\equiv p \wedge \sim q$ Double Negation Law

(ii) $\sim(p \vee \sim(p \wedge q))$
$\equiv \sim p \wedge \sim(\sim(p \wedge q))$ De Morgan's Law
$\equiv \sim p \wedge (p \wedge q)$ Double Negation Law
$\equiv (\sim p \wedge p) \wedge q$ Associative Law
$\equiv F \wedge q$ Contradiction Law
$\equiv F$ Domination Law and Commutative Law.

(iii) $\sim (p \vee (\sim p \wedge q))$

$\equiv \sim p \wedge \sim (\sim p \wedge q)$ De Morgan's Law

$\equiv \sim p \wedge (\sim\sim p \vee \sim q)$ De Morgan's Law

$\equiv \sim p \wedge (p \vee \sim q)$ Double Negation Law

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ Distribution Law

$\equiv (p \wedge \sim p) \vee (\sim p \wedge \sim q)$ Commutative Law

$\equiv F \vee (\sim p \wedge \sim q)$ Contradiction Law

$\equiv (\sim p \wedge \sim q) \vee F$ Commutative Law

$\equiv (\sim p \wedge \sim q)$ Identity Law

(iv) $p \vee (p \wedge q)$
$\equiv (p \wedge T) \vee (p \wedge q)$ Identity (in reverse)
$\equiv p \wedge (T \vee q)$ Distributive (in reverse)
$\equiv p \wedge T$ Domination
$\equiv p$ Identity

Example 1.4.3. Find a simple form for the negation of the proposition
"If the sun is shining, then I am going to the ball game."
Solution.

This proposition is of the form $p \rightarrow q$. Since $\sim(p \rightarrow q) \equiv \sim(\sim p \vee q) \equiv (p \wedge \sim q)$. This is the proposition "The sun is shining, and I am not going to the ball game."