1.5. Normal or Canonical Forms

From Examples 1.4.1.(ii)(iii), even though are expressed with only ∧, ∨ and ~, it is still hard to tell without doing a proof.

(i) What we need is a unique representation of a compound proposition that uses ∧, ∨ and ~.

(ii) This unique representation is called the Disjunctive Normal Form as define below.

Definition 1.5.1. A clause that contains only ∨ is called a **disjunctive clause** and only ∧ is called a **conjunctive clause**.

Negation is allowed, but only directly on variables.

Example 1.5.2.

(i) p∨~q∨r : a disjunctive clause.

(ii) ~p∧q∧~r : a conjunctive clause.

(iii) ~p∧~q∨r : neither.

Definition 1.5.3.

(i) A bunch of disjunctive clauses together with ∧, it is called **conjunctive normal form(CNF)**.

(i) A bunch of conjunctive clauses together with ∨, it is called **disjunctive normal form(DNF)**.

Remark 1.5.4. The individual conjunction clauses (disjunctive clauses) that make up the DNF (CNF) are called **minterms**.

Example 1.5.5.

(i) p : Both normal forms, since single literal is always conjunctive and disjunctive normal form.

(ii) p ∧ q : Conjunctive and disjunctive normal form.

(iii) (p∧q∧~r ∧s)∨(~q∧s)∨(p∧s): disjunctive normal form.

(iv) (p∨q∨~r∨s) ∧ (~q∨s)∧ ~s : conjunctive normal form.

(v) (p∨r)∧(q∧(p∨~q)) : not in a normal form.