

(vi)  $\sim p \vee q \vee r$  and  $\sim p \wedge q \wedge r$  : Both normal forms.

**Remark 1.5.6.** It turns out we can turn any logical proposition into either normal form.

(i) We can use the definitions to get rid of  $\rightarrow$ ,  $\leftrightarrow$ , and  $\underline{\vee}$ .

(ii) Use De Morgan's laws to move any  $\sim$  in past original statement, so they sit on the variables.

(iii) Use double negation to get rid of any  $\sim\sim$  that showed up.

(iv) Use the distributive rules to move things in/out of original statement as we need to.

**Example 1.5.7.** Converting  $\sim((\sim p \rightarrow \sim q) \wedge \sim r)$  to conjunctive normal form.

$$\begin{aligned}
 \sim((\sim p \rightarrow \sim q) \wedge \sim r) &\equiv \sim((\sim\sim p \vee \sim q) \wedge \sim r) && \text{Definition} \\
 &\equiv \sim((p \vee \sim q) \wedge \sim r) && \text{Double negation} \\
 &\equiv \sim(p \vee \sim q) \vee \sim\sim r && \text{De Morgan's} \\
 &\equiv \sim(p \vee \sim q) \vee r && \text{Double negation} \\
 &\equiv (\sim p \wedge \sim\sim q) \vee r && \text{De Morgan's} \\
 &\equiv (\sim p \wedge q) \vee r && \text{Double negation : disjunctive normal form} \\
 &\equiv (\sim p \vee r) \wedge (q \vee r) && \text{Distributive : conjunctive normal form}
 \end{aligned}$$

It was actually in disjunctive normal form in the second-last step.