

The disjunctive normal form for  $(p \rightarrow q) \wedge \sim r$  is

$$(p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r).$$

**Remark 1.6.2.** If we want to get the conjunctive normal form of a logical proposition, construct

- (1) the disjunctive normal form of its negation,
- (2) negate again and apply De Morgan's Law.

**Example 1.6.3.** Find the conjunctive normal form of the logical proposition

$$(p \wedge \sim q) \vee r.$$

**Solution.**

(1) Negate:  $\sim[(p \wedge \sim q) \vee r] \equiv (\sim p \vee q) \wedge \sim r.$

(2) Find the disjunctive normal form of  $(\sim p \vee q) \wedge \sim r.$

p	q	r	$\sim p$	$\sim r$	$\sim p \vee q$	$(\sim p \vee q) \wedge \sim r$	
T	T	T	F	F	T	F	
T	T	F	F	T	T	T	←
T	F	T	F	F	F	F	
T	F	F	F	T	F	F	
F	T	T	T	F	T	F	
F	T	F	T	T	T	T	←
F	F	T	T	F	T	F	
F	F	F	T	T	T	T	←

The disjunctive normal form for  $(\sim p \vee q) \wedge \sim r$  is

$$(p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r).$$

(3) The conjunctive normal form for  $(p \wedge \sim q) \vee r$  is then the negation of this last expression, which, by De Morgan's Laws, is

$$(\sim p \vee \sim q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee r).$$

**Remark 1.6.4.**

(1)  $p \vee q$  can be written in terms of  $\wedge$  and  $\sim$ .

(2) We can write every compound logical proposition in terms of  $\wedge$  and  $\sim$ .