

Definition 1.7.5.

The statement $q \rightarrow p$ is called the **converse** of the statement $p \rightarrow q$ and the statement $\sim p \rightarrow \sim q$ is called the **inverse**.

Generally, the statement and its converse not necessary equivalent. Therefore, $p \Rightarrow q$ does not mean that $q \Rightarrow p$.

Example 1.7.6. The statement “the triangle which has equal sides, has two equal legs” equivalent to the statement “ the triangle which has not two equal legs has no equal sides”.

1.8. Quantifiers

Recall that a formula is a statement whose truth value may depend on the values of some variables. For example,

“ $x \leq 5 \wedge x > 3$ ” is true for $x = 4$ and false for $x = 6$.

Compare this with the statement

"For every x , $x \leq 5 \wedge x > 3$," which is definitely false and the statement

"There exists an x such that $x \leq 5 \wedge x > 3$," which is definitely true.

Definition 1.8.1.

(i) The phrase "for all x " ("for every x ", "for each x ") is called a **universal quantifier** and is denoted by $\forall x$.

(ii) The phrase "for some x " ("there exists an x ") is called an **existential quantifier** and is denoted by $\exists x$.

(iii) A formula that contains variables is not simply true or false unless each of these variables is **bound** by a quantifier.

(iv) If a variable is not bound the truth of the formula is contingent on the value assigned to the variable from the universe of discourse.

Definition 1.8.2. (The Universal Quantifier)