In words, this is "~r: There exists a real number whose square is negative".

(ii) r: For all x, there exists y such that $xy = 1$.
Solution.
r: $\forall\, x, \exists\, y$ such that $xy = 1$.
~r: $\sim (\forall\, x, \exists\, y$ such that $xy = 1) \equiv \exists x, \forall\, y$ such that $\sim (xy = 1) \equiv \exists x, \forall\, y$ such that $xy \neq 1$.
In words, this is "~r: There exists x for all y such that $xy \neq 1$".
(iii) p: student who is intelligent will succeed.
Solution.
Let r: student who is intelligent.
s: succeed.
p: r → s
~p: ~ (r → s) ≡~ (~ r ∨ s) Implication Low.

≡ r ∧~ s. De Morgan's Law
~p: student who is intelligent will not succeed.

There are six ways in which the quantifiers can be combined when two variables are present:

(1) $\forall\, x\, \forall\, y\, f(x, y)= \forall\, y\, \forall\, x\, f(x, y)$=For every x, for every y $f(x, y)$.

(2) $\forall\, x\, \exists\, y\, f(x, y)$= For every x, there exists a y such that $f(x, y)$.

(3) $\forall\, y\, \exists\, x\, f(x, y)$= For every y, there exists an x such that $f(x, y)$.

(4) $\exists\, x\, \forall\, y\, f(x, y)$= There exists an x such that for every y $f(x, y)$.

(5) $\exists\, y\, \forall\, x\, f(x, y)$= There exists a y such that for every y $f(x, y)$.

(6) $\exists\, x\, \exists\, y\, f(x, y)= \exists\, y\, \exists\, x\, f(x, y)$= There exists an x such that there exists a y $f(x, y)$.

Example 1.8.10. Show that the following are equivalents.

(i) $\sim[\forall\, x\, \forall\, y\, f(x, y)] \equiv \exists x\ \exists y \sim f(x, y)$.

(ii) $\sim[\exists\, x\, \forall\, \exists\, f(x, y)] \equiv \forall x\ \forall y \sim f(x, y)$.

(iii) $\sim[\forall\, x\, \exists\, y\, f(x, y)] \equiv \exists x\ \forall y \sim f(x, y)$.