

(iii)  $\sim[\exists x \forall y f(x, y)] \equiv \forall x \exists y \sim f(x, y)$ .

**Solution. Exercise.**

## 1.9. Logical Reasoning

### Definition 1.9.1. (Arguments)

An **argument** is a series of statements starting from hypothesis (premises) and ending with the conclusion.

From the definition, an argument might be valid or invalid.

### Definition 1.19.2. (Valid Arguments)

An argument is said to be **valid** if the hypothesis implies the conclusion; that is, if  $s$  is a statement implies from the statements  $s_1, s_2, \dots, s_n$ , then write as

$$s_1, s_2, \dots, s_n \mapsto s.$$

### Example 1.9.3.

(i) Let  $s_1$ : Some mathematicians are engineering

$s_2$ : Ali is mathematician

$s$ : Ali is engineering

### Solution.

The argument  $s_1, s_2 \mapsto s$  is not valid, since not all mathematicians are engineering.

(ii) Let  $s_1$ : There is no lazy student

$s_2$ : Ali is artist

$s_3$ : All artist are lazy

Find a conclusion  $s$  for the above premises making the argument  $s_1, s_2, s_3 \mapsto s$  is valid.

### Solution.

Ali is

### Remark 1.9.4.

(i) An argument

$$s_1, s_2, \dots, s_n \mapsto s$$

is valid if and only if

$$(s_1 \wedge s_2 \wedge \dots \wedge s_n) \rightarrow s$$

is tautology; that is,

$$(s_1 \wedge s_2 \wedge \dots \wedge s_n) \Rightarrow s.$$

Also, any valid argument called **the proof**.