q: a is an even number.
Then
~p: $a^2$is an odd number,
~q: a is an even number.
Therefore, The contrpositive statement is
a is an odd number → a^2 is an odd number.
(1) $a = 2(k + 1)$, k is an integer (definition of odd number).
(2) $a^2 = 4k^2 + 4k + 1$, square both sides of (1)
(3) $a^2 = 2(2k^2 + 2k) + 1$,
(4) a^2 is odd number, since $2k^2 + 2k$ is an integer and definition of odd number.

1.10.4 To Prove Statement of Type (p ⟷ q).
(i) Since (p ⟶ q) ∧ (q ⟶ p) ≡ (p ⟷ q), so we can proved first p ⟶ q and then proved q ⟶ p.
(ii) Moved from p into q through series of logical equivalent statements s_i as follows:

$$p \leftrightarrow s_1$$
$$s_1 \leftrightarrow s_2$$
$$\vdots$$
$$s_n \leftrightarrow q$$

This is exactly the tautology

$$((p \leftrightarrow s_1) \wedge (s_1 \leftrightarrow s_2) \wedge \dots \wedge (s_n \leftrightarrow q)) \rightarrow (p \leftrightarrow q).$$

1.10.5 To Prove Statement of Type ∀x $P(x)$ or ∃x $P(x)$.
(i) To prove a sentence of type ∀ x $P(x)$, we suppose x is an arbitrary element and then prove that $P(x)$ is true.
(ii) To prove a sentence of type ∃ x $P(x)$, we have to prove there exist at least one element x such that $P(x)$ is true.

1.10.6 To Prove Statement of Type p ∨ r ⟶ q.
Depending on the tautology

$$[(p\rightarrow q)\wedge(r\rightarrow q)] \rightarrow[(p\vee r)\rightarrow q]$$

We must prove that p⟶q and r⟶q.

Example 1.10.7. Prove that

$$(a = 0 \vee b = 0) \rightarrow ab = 0$$

where a, b are real numbers.
Proof.
Firstly, we prove that $a = 0 \rightarrow ab = 0$.
Suppose that $a = 0$, then $ab = 0. b = 0$.