

Secondly, we prove that $b = 0 \rightarrow ab = 0$.

Suppose that $b = 0$, then $ab = a \cdot 0 = 0$.

Therefore, the statement $(a = 0 \vee b = 0) \rightarrow ab = 0$ is tautology.

1.10.8 . Proof by Contradiction.

The contradiction is always false statement whatever the truth values of its components. Proof by contradiction is type of indirect proof.

The way of proof logical proposition **p** by contradiction start by supposing that $\sim p$ and then try to find sentence of type

$$R \wedge \sim R$$

where R is any sentence contain **p** or any pervious theorem or any axioms or any logical propositions.

This way supports by the tautology

$$\sim [\sim p \wedge (R \wedge \sim R)] \rightarrow p.$$

By this way we can also prove sentences of type $\forall x P(x)$ or $\exists x P(x)$ or $(p \rightarrow q)$ or $(p \Rightarrow q)$.

Example 1.10.9. Prove that $x \neq 0 \Rightarrow x^{-1} \neq 0$, x is real number.

Proof.

Let $p: x \neq 0$,

$q: x^{-1} \neq 0$.

We must prove $p \Rightarrow q$.

Suppose $\sim(p \Rightarrow q)$ is true.

(1) $\sim(p \rightarrow q)$ is tautology, by def. of logical implication.

(2) $p \wedge \sim q$ is tautology, since $\sim(p \rightarrow q) \equiv p \wedge \sim q$

(3) $x \neq 0 \wedge x^{-1} = 0$.

(4) $x \cdot x^{-1} = 1 \neq 0$.

(5) $x \cdot x^{-1} = x \cdot 0 = 0$.

(6) $1 = 0$, from (4) and (5). This is contradiction, since $1 \neq 0 \wedge 1 = 0$.

Thus, the statement $\sim(p \Rightarrow q)$ is not true. Therefore, $p \Rightarrow q$.