

AIR POLLUTANT CONCENTRATION MODELS

INTRODUCTION

Atmospheric dispersion modelling is the mathematical simulation of how air pollutants disperse in the ambient atmosphere. The dispersion models are used to estimate or to predict the downwind concentration of air pollutants or toxins emitted from sources such as industrial plants, vehicular traffic or accidental chemical releases. The perfect air concentration model will estimate the concentration emitted from any specified set of pollutant, at any location, at any period of time and with high confidence levels in our prediction. The models are typically employed to determine whether the ambient air is in compliance with the Malaysia Ambient Air Quality Guideline (MAAQG).

Practically, it is performed using computer programs that solved the mathematical equations and algorithms which simulate the pollutant dispersion. Such models are important especially to governmental agencies tasked with protecting and managing the ambient air quality. The models also supplied information to assist in the design of effective control strategies to reduce emissions of harmful air pollutants.

Air dispersion models are also used for emergency planning of accidental chemical releases. Models are used to determine the consequences of accidental releases of hazardous or toxic materials. Accidental releases may result in fires, spills or explosions that involve hazardous materials, such as chemicals or radionuclides. The results of dispersion modeling, using worst case accidental release source terms and meteorological conditions, can provide an estimation of location impacted areas and ambient concentrations. Thus, it and can be used to determine protective actions appropriate to the accident. In this chapter we will learn from the simplest model (least reliable) to the most complex (most reliable) model so that the theories behind the mathematical simulation program can be revealed.

All the models studied here will be using a general material balance that is (Equation 1):

$$\begin{array}{rcccl} \boxed{\text{Accumulation}} & = & \boxed{\text{All flow rates}} & - & \boxed{\text{All flow rates}} \\ \boxed{\text{rate}} & & \boxed{\text{in}} & & \boxed{\text{out}} \\ & & & & \\ & + & \boxed{\text{Creation}} & - & \boxed{\text{Destruction}} \\ & & \boxed{\text{rate}} & & \boxed{\text{rate}} \end{array}$$

Equation 1

FIXED BOX MODEL

The box model is the simplest of the model types (Atkins,...). It assumes the volume of atmospheric air is in the shape of a box. It also assumes that the air pollutants inside the box are homogeneously distributed. Although useful, this model is very limited in its ability to accurately predict dispersion of air pollutants because the assumption of homogeneous pollutant distribution is much too simple. To start adopting box model, the following major simplifying assumptions must be made according to the illustrated figure below (Figure 1):

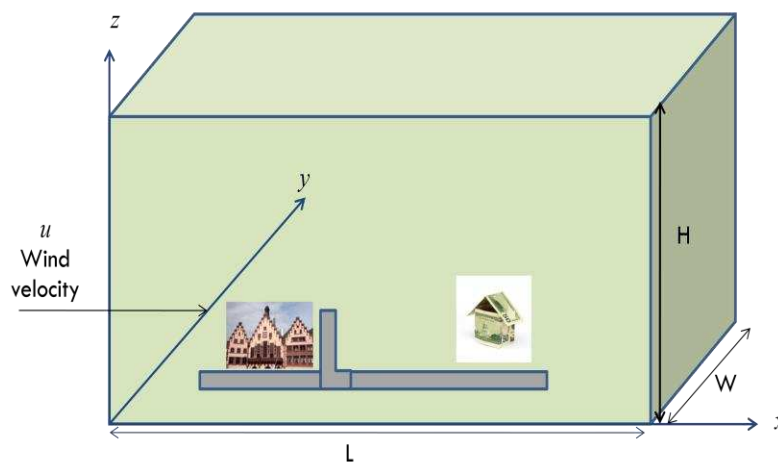


Figure 1: Rectangular city, showing the symbol of used in the fixed box model

- I. The city is a rectangle with dimension W and L and one side is parallel with wind direction.
- II. Complete atmospheric turbulence is produced and total mixing of pollutants up to H and no mixing above this height.
- III. The turbulence is strong enough in the upwind direction that the pollutant concentration is uniform in the whole volume of air over the city and not higher at downwind side than upwind side.*
- IV. The wind blows in x direction with velocity u. This velocity is constant and is independent of time, location or elevation above the ground (steady state condition). We use average u between at the ground and at H.
- V. The concentration of pollutant entering the city (at x = 0) is constant and is equal to b (background concentration). The units are g/m³ or µg/m³.
- VI. The air pollution rate of the city is Q (g/s). This is usually given as emission rate per unit area, q, g/s.m². The conversion can be made by

$$Q = qA$$

where A is the area (W x L).

- VII. No pollutant enters or leaves through the top or side of the box.
- VIII. The pollutant is long-lived in the atmosphere (destruction rate = 0).

*This emission rate is constant and unchanging with time (in steady state condition)

With all these assumption, the general mass balance equation is simplified as below (Equation 2):

$$\begin{array}{r}
 \begin{array}{|c|} \hline \text{Accumulation} \\ \text{rate} \\ \hline \end{array} \begin{array}{l} \nearrow \\ 0 \end{array} = \begin{array}{|c|} \hline \text{All flow rates} \\ \text{in} \\ \hline \end{array} - \begin{array}{|c|} \hline \text{All flow rates} \\ \text{out} \\ \hline \end{array} \\
 + \begin{array}{|c|} \hline \text{Creation rate} \\ \hline \end{array} \begin{array}{l} \nearrow \\ 0 \end{array} - \begin{array}{|c|} \hline \text{Destruction} \\ \text{rate} \\ \hline \end{array} \begin{array}{l} \nearrow \\ 0 \end{array}
 \end{array}$$

So, that the equation is simplified as below;

$$0 = (\text{all flow rates in}) - (\text{all flow rates out})$$

Equation 2

There are 2 flow rates. The flow rate of the pollutant into the upwind side is

$$\begin{aligned} \text{Flow rate in} &= (uWH)b = (\text{volume/time}) \times (\text{mass/volume}) \\ &= \text{mass/time} \end{aligned}$$

The second flow rate in is that the pollutant emitted by the city into the lower boundary,

$$\text{Flow rate in} = Q = qWL$$

According to the assumption, the concentration of entire city is a constant and equal to c .
The flow rate out is:

$$\text{Flow rate out} = (uWH)c$$

Then, substitute all these equations into equation 2, will yield:

$$0 = (uWHb + qWL) - uWHc$$

$$0 = [W(uHb + qL) - uHc]$$

$$uHc = uHb + qL$$

$$\therefore c = b + \frac{qL}{uH}$$

Equation 3

This equation is the same as the continuous-stirred-tank reactor (CSTR) model that is widely used in chemical engineering.

Example 1:

A city with dimension $W \times L \times H$ (7 km x 13 km x 1.5 km) had a wind velocity of 4 m/s. The upwind concentration of SO_2 is = $10 \mu\text{g}/\text{m}^3$. The emission rate for the city is $4.5 \times 10^6 \text{ g}/\text{s} \cdot \text{m}^2$.
What is the concentration of SO_2 over the city?

Solution:

$W = 7 \text{ km};$

$L = 13 \text{ km}; H = 1.5 \text{ km}$

$u = 4 \text{ m/s};$

$b = 10 \text{ } \mu\text{g/m}^3; q = 4.5 \times 10^{-6} \text{ g/s.m}^2$

$$c = b + \frac{qL}{uH}$$

$$c = \frac{10 \mu\text{g}}{\text{m}^3} + \left(4.5 \times 10^{-6} \frac{\text{g}}{\text{s.m}^2} \right) \left(\frac{13000\text{m}}{(4\text{m/s})(1500\text{m})} \right)$$

$$c = \frac{10 \mu\text{g}}{\text{m}^3} + \frac{9.75 \times 10^{-6} \text{ g}}{\text{m}^3}$$

$$c = \frac{10 \mu\text{g}}{\text{m}^3} + \frac{9.75 \mu\text{g}}{\text{m}^3} = 19.75 \frac{\mu\text{g}}{\text{m}^3}$$

DO IT YOURSELF:

Name of strip	Length, km	Emission rate, q, g/s.km ²	Mixing height H, m
Upwind suburbs	5	100	400
Downtown	2	500	500
Downwind suburbs	5	100	400

Estimate the concentration of carbon monoxide at the downwind edge of a city. The city may be considered to consist to 3 parallel strips, located perpendicular to the wind. For all the strips the wind velocity is 3 m/s. the properties of each of the strips are described in following table. Assume that the fixed-box model applies to each of the strips. The background concentration b in the air entering the upwind suburbs is 1 mg/m^3 .

(Answer: $1.5 \times 10^{-3} \text{ g/m}^3$ or 1.5 mg/m^3)

The model indicates that the upwind concentration for a long lived pollutant is additive to the concentration produced by the city. Clearly from equation 3, the concentration, c increases with the increase of q and L and decreases with the increases in u and H .

Most of the models predict concentrations for only one specific meteorological condition. To calculate the annual average concentration of some pollutant, we need to calculate the concentration for each meteorological condition and then multiply by the frequency of occurrence for that meteorological condition.

$$\left(\begin{array}{c} \text{Annual} \\ \text{average} \\ \text{concentration} \end{array} \right) = \sum_{\text{over all meteorologies}} \left(\begin{array}{c} \text{concentration} \\ \text{for} \\ \text{that} \\ \text{meteorology} \end{array} \right) \left(\begin{array}{c} \text{frequency} \\ \text{of} \\ \text{occurrence} \\ \text{of} \\ \text{that} \\ \text{meteorology} \end{array} \right) \quad \text{Equation 4}$$

Example 2:

For the city in Example 1, the meteorological conditions described ($u = 4\text{m/s}$, $H = 1.5\text{ km}$) occur 34% of the time. For the remaining of time, the wind blows from L direction shown in figure 1 at velocity of 8 m/s and the same mixing height. What is the annual concentration of CO in this city?

Solution:

1) Find the concentration for other meteorology:

$$c = b + \frac{qL}{uH}$$

$$c = \frac{10\mu\text{g}}{\text{m}^3} + \left(4.5 \times 10^{-6} \frac{\text{g}}{\text{s.m}^2} \right) \left(\frac{7000\text{m}}{(8\text{m/s})(1500\text{m})} \right)$$

$$c = \frac{10\mu\text{g}}{\text{m}^3} + \frac{2.625\mu\text{g}}{\text{m}^3} = 12.625 \frac{\mu\text{g}}{\text{m}^3}$$

Using equation 4, calculate the annual average concentration of CO.

$$\left(\begin{array}{l} \text{Annual} \\ \text{average} \\ \text{concentration} \end{array} \right) = \left(19.75 \frac{\mu\text{g}}{\text{m}^3} \times 0.34 \right) + \left(12.625 \frac{\mu\text{g}}{\text{m}^3} \times 0.66 \right)$$

$$\left(\begin{array}{l} \text{Annual} \\ \text{average} \\ \text{concentration} \end{array} \right) = 15.05 \frac{\mu\text{g}}{\text{m}^3}$$

GAUSSIAN PLUME MODEL

Gaussian plume model is usually called as diffusion model and sometimes dispersion model. Most of diffusion model used Gaussian plume idea which is also a material balance model. In this model, a point source such as a factory smoke stack is selected and the downwind concentration resulting from this point source is calculated.

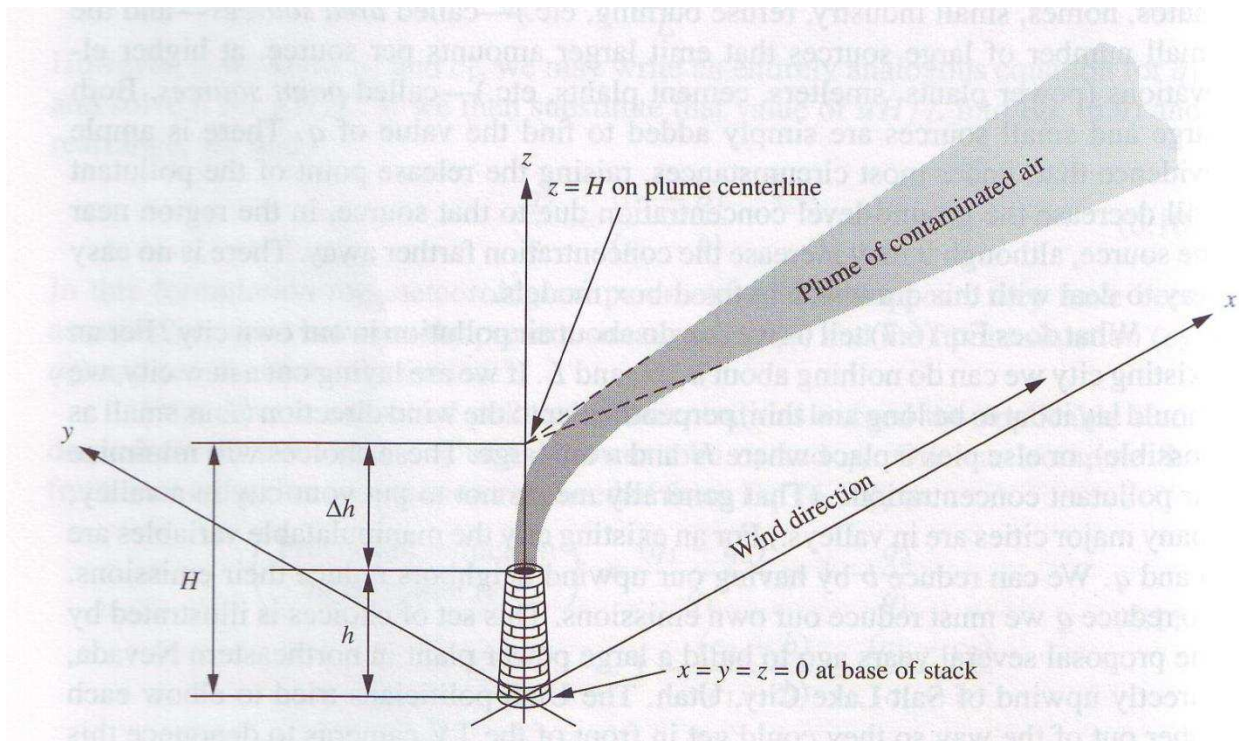


Figure 2: Coordinate system and nomenclature for Gaussian Plume Idea

From the figure, the origin of the coordinate system is placed at the base of the smokestack with the x axis aligned in the downwind direction. The contaminated gas stream or plume rising from the smoke stack (Δh) and then travel in x direction and spread in the y and z direction as it travels.

The plumes normally rise higher above the smoke stack because they are emitted at higher temperature than atmosphere and with vertical velocity. For the calculations, the plume is assumed to be emitted at coordinate; $(0, 0, H)$ where H is effective stack height which are the summation of the physical stack height and plume rise ($h + \Delta h$). The smoke emitted at the point source is assumed to be a non-buoyant pollutant at emission rate Q (g/s) and blows in x direction with velocity u that independent of time, elevation or location.

The problem is to compute the concentration due to this source at any point (x, y, z) for $x > 0$. If the molecular diffusion alone were causing the plume to mix with the surrounding air, the plume will spread slowly as a thin streak moving straight down the sky.

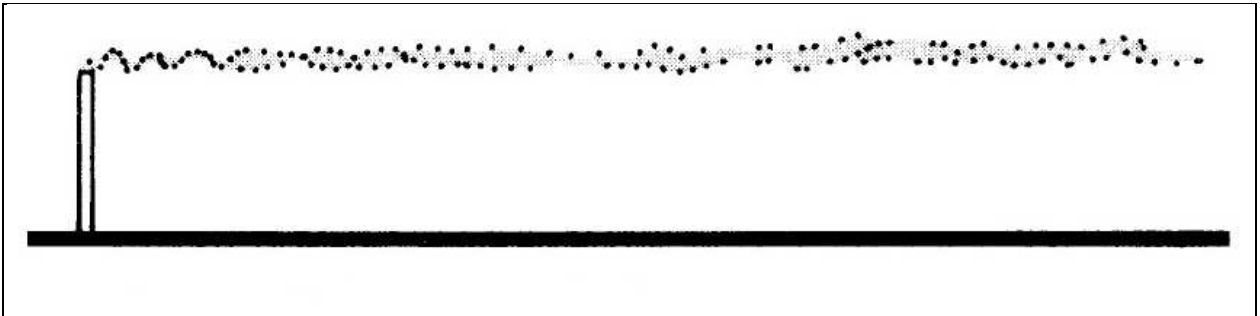


Figure 3: Time exposure of a visible plume

The spread of the plume does not depend on molecular diffusion only. The main cause of the spreading is the large scale turbulent mixing that exists in atmosphere. The twisting behaviour is caused by the turbulent motion of the atmosphere that is superimposed on the plume's large scale linear motion caused by horizontal wind. This turbulent motion is random in nature; a snapshot taken after a few minutes after the first will show different snapshot in different places. However, time averages the variation and thus the concentration will appear quit uniform and symmetrical.

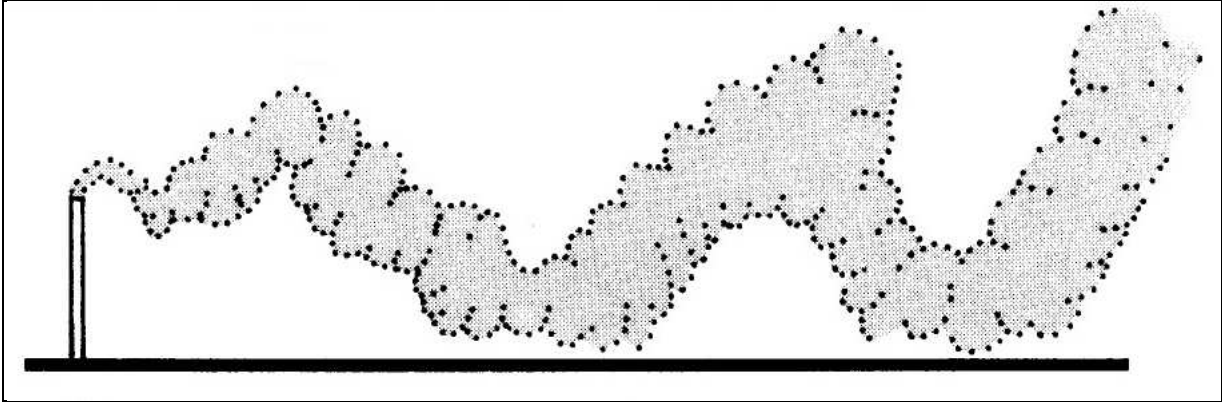


Figure 4: Snapshot of a visible plume

Gaussian Plume Derivation

Discovering on how the contaminated air expands by turbulent mixing, we need to perform material balance around some small cube of space near the centre of the plume. Assume that neither material is created nor destroyed in the atmosphere;

$$\begin{array}{rcccl}
 \boxed{\text{Accumulation}} & = & \boxed{\text{All flow rates}} & - & \boxed{\text{All flow rates}} \\
 \boxed{\text{rate}} & & \boxed{\text{in}} & & \boxed{\text{out}} \\
 & & & & \\
 + & & \boxed{\text{Creation rate}} & - & \boxed{\text{Destruction rate}} \\
 & & \nearrow 0 & & \nearrow 0
 \end{array}$$

$$\text{Accumulation rate} = (\text{all flow rates in}) - (\text{all flow rates out})$$

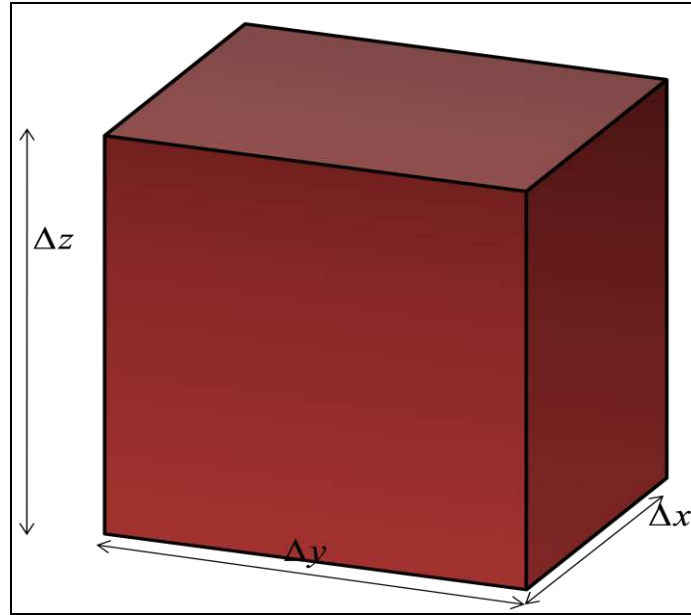


Figure 5: Dimension of cube used for material balance

The accumulation rate is the time derivative of the amount contained that is concentration and volume. But the volume of the cube is not changing with time;

$$\text{Accumulation rate} = \frac{\partial}{\partial t}(cV) = V \frac{\partial c}{\partial t} = \Delta x \Delta y \Delta z \frac{\partial c}{\partial t}$$

Equation 5

There will be no bulk flow into or out of the cube because the cube is moving with the local wind velocity. However, there are flows through all the surfaces of the cube due to turbulent mixing.

$$\text{Flux} = (\text{time rate of mass flow/unit area}) = -K \frac{\partial c}{\partial n}$$

Equation 6

where;

c = concentration

n = distance in the direction considered (normally x, y or z)

K = turbulent dispersion coefficient

The units of the flux is mass/time . area (e.g: g/s.m²) and $\partial c / \partial n$ has dimension of mass/time⁴. To obtain the flux, the K value must be length²/time. The negative sign indicate that the flow is from higher concentration to low.

This approximation in Equation 6 is called the gradient transport or *K theory* or first order closure approach and *K* is called eddy diffusivity. The cube has 2 faces that loc Equation 7 . So, the net flow into the cubes in the x direction is (Equation 7);

$$\text{(Net flow into the cube in the x direction)} = \left[\left(\frac{-K\partial c}{\partial x} \right)_{at;x} - \left(\frac{-K\partial c}{\partial x} \right)_{at;x+\Delta x} \right] \Delta y \Delta z \quad \text{Equation 7}$$

where the first term represent the flow in whereas the second term represents flow out.

By the same procedure we can write for axis y (Equation 8) and z (Equation 9).

$$\text{(Net flow into the cube in the y direction)} = \left[\left(\frac{-K\partial c}{\partial y} \right)_{at;y} - \left(\frac{-K\partial c}{\partial y} \right)_{at;y+\Delta y} \right] \Delta x \Delta z \quad \text{Equation 8}$$

$$\text{(Net flow into the cube in the z direction)} = \left[\left(\frac{-K\partial c}{\partial z} \right)_{at;z} - \left(\frac{-K\partial c}{\partial z} \right)_{at;z+\Delta z} \right] \Delta x \Delta y \quad \text{Equation 9}$$

These equations represent the flows in or out through the 6 faces by turbulent mixing. From the simplified material balance, we know that the sum of **equation 6** to **8** is equal to accumulation

$$\frac{\partial c}{\partial t} = \left[\frac{\left(\frac{K\partial c}{\partial x} \right)_{at;x+\Delta x} - \left(\frac{K\partial c}{\partial x} \right)_{at;x}}{\Delta x \Delta y \Delta z} \right] \Delta y \Delta z + \left[\frac{\left(\frac{K\partial c}{\partial y} \right)_{at;y+\Delta y} - \left(\frac{K\partial c}{\partial y} \right)_{at;y}}{\Delta x \Delta y \Delta z} \right] \Delta x \Delta z + \left[\frac{\left(\frac{K\partial c}{\partial z} \right)_{at;z+\Delta z} - \left(\frac{K\partial c}{\partial z} \right)_{at;z}}{\Delta x \Delta y \Delta z} \right] \Delta y \Delta x \quad \text{Equation 10}$$

rate. We now substitute **equation 8** to **9** into the material balance and divided with $\Delta x \Delta y \Delta z$.

$$\frac{\partial c}{\partial t} = \left[\frac{\left(\frac{K\partial c}{\partial x} \right)_{at;x+\Delta x} - \left(\frac{K\partial c}{\partial x} \right)_{at;x}}{\Delta x} \right] + \left[\frac{\left(\frac{K\partial c}{\partial y} \right)_{at;y+\Delta y} - \left(\frac{K\partial c}{\partial y} \right)_{at;y}}{\Delta y} \right] + \left[\frac{\left(\frac{K\partial c}{\partial z} \right)_{at;z+\Delta z} - \left(\frac{K\partial c}{\partial z} \right)_{at;z}}{\Delta z} \right] \quad \text{Equation 11}$$

But

$$\lim_{\Delta x \rightarrow 0} \frac{\left(\frac{K\partial c}{\partial x} \right)_{at;x+\Delta x} - \left(\frac{K\partial c}{\partial x} \right)_{at;x}}{\Delta x} = \frac{K\partial^2 c}{\partial x^2} \quad \text{Equation 12}$$

so that if we take the limit on an infinitesimally small cube **equation 10** become:

$$\frac{\partial c}{\partial t} = K \frac{\partial^2 c}{\partial x^2} + K \frac{\partial^2 c}{\partial y^2} + K \frac{\partial^2 c}{\partial z^2} \quad \text{Equation 13}$$

For **equation 13**, we cannot factor the K because of experimental data indicate that for turbulent diffusion in atmosphere the values of K in 3 directions are not the same. So, we will write the 3 Ks as K_x , K_y and K_z .

$$\frac{\partial c}{\partial t} = K_x \frac{\partial^2 c}{\partial x^2} + K_y \frac{\partial^2 c}{\partial y^2} + K_z \frac{\partial^2 c}{\partial z^2} \quad \text{Equation 14}$$

The Gaussian plume equation (**equation 14**) is regularly applied to pollutant spreading in 1(x) dimension, 2 (x and y) dimension or 3 (x, y and z) dimensions. The concentrations for 1, 2 and 3 dimensional spreading are;

$$c = \frac{X}{2(\pi t)^{1/2} K_x^{1/2}} \exp \left[- \left(\frac{1}{4t} \right) \left(\frac{x^2}{K_x} \right) \right] \quad \text{For 1 dimension} \quad \text{(Equation 15)}$$

$$c = \frac{X}{4(\pi t)(K_x K_y)^{1/2}} \exp \left[- \left(\frac{1}{4t} \right) \left(\frac{x^2}{K_x} + \frac{y^2}{K_y} \right) \right] \quad \text{For 2 dimensions} \quad \text{(Equation 16)}$$

$$c = \frac{X}{8(\pi t)^{3/2} (K_x K_y K_z)^{1/2}} \exp \left[- \left(\frac{1}{4t} \right) \left(\frac{x^2}{K_x} + \frac{y^2}{K_y} + \frac{z^2}{K_z} \right) \right] \quad \text{For 3 dimensions} \quad (\text{Equation 17})$$

Gaussian Plume, 2 Dimensional Spreading

In order to use the 2 dimensions equation (**equation 16**) for solving problems, some material balances and substitutions had been made to obtain:

$$c = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp - \left[\frac{y^2}{2\sigma_y^2} + \frac{(z-H)^2}{2\sigma_z^2} \right] \quad \text{Basic Gaussian plume equation} \quad (\text{Equation 18})$$

where σ_y and σ_z are called horizontal and vertical dispersion coefficients. The unit is usually in meters (m). If we set $y = (z-H) = 0$, then the exponential parts will be exponent 0 that is 1.

The first term is the concentration on the centreline of the plume. The two values increase with downwind distance, so that this centreline concentrations decrease with the downwind distance. The second and third terms show how the concentration decreases as we move horizontal and vertically.

Example:

A factory emits 34 g/s of SO₂ at H height. The wind speed is 5 m/s. At a distance of 1 km downwind, the values of σ_y and σ_z are 24 m and 37 m, respectively. What are the concentration of SO₂ at the centreline of the plume, and at a point 60 m to the side and 20 m below the centreline?

Solution:

At centreline ($z=H$), so;

$$c = \frac{Q}{2\pi u \sigma_y \sigma_z} = \frac{34 \text{ g/s}}{2\pi(5 \text{ m/s})(24 \text{ m/s})(37 \text{ m/s})} = 0.00122 = 1220 \frac{\mu\text{g}}{\text{m}^3}$$

At the point away from the centreline, we must calculate the 2nd and 3rd item;

$$\exp\left[-\frac{y^2}{2\sigma_y^2} + \frac{(z-H)^2}{2\sigma_z^2}\right] = \exp\left[-\frac{1}{2}\left(\frac{60\text{m}}{24\text{m}}\right)^2 + \frac{1}{2}\left(\frac{-20\text{m}}{37\text{m}}\right)^2\right] = 0.038$$

So, the concentration at $y=60$ and $z=-20$ is;

$$c = \left(1220 \frac{\mu\text{g}}{\text{m}^3}\right)(0.038) = 46 \frac{\mu\text{g}}{\text{m}^3}$$

The basic Gaussian plume equation predicts a plume that is symmetrical with respect to y and z and ignoring the ground level effect (H).

Different values of σ_y and σ_z mean that the spreading in the vertical and horizontal directions is not equal. Usually the values of $\sigma_y > \sigma_z$. The horizontal dispersion coefficient forms a family of a straight line (for various atmospheric conditions) with slope of 0.894.

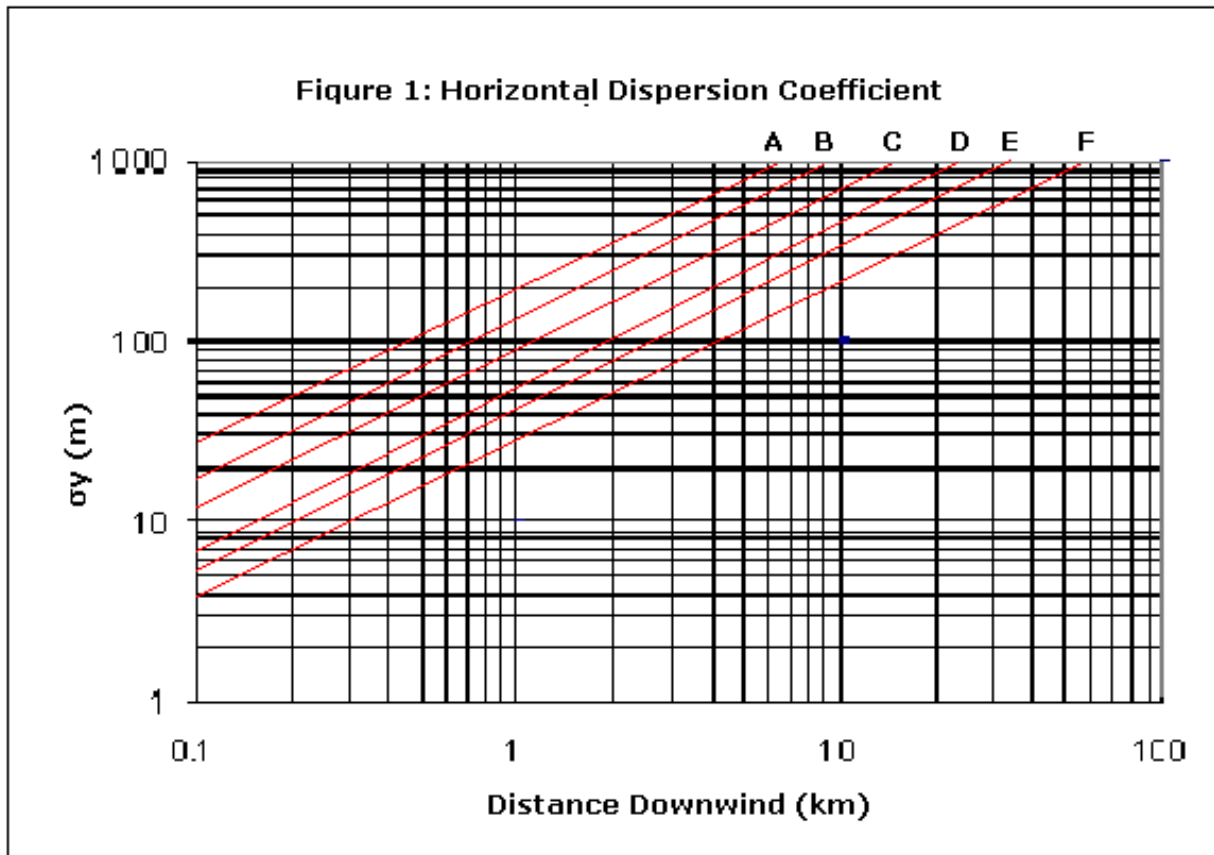


Figure 6: Horizontal dispersion coefficient σ_y as a function of downwind distance from the source for various stability categories.

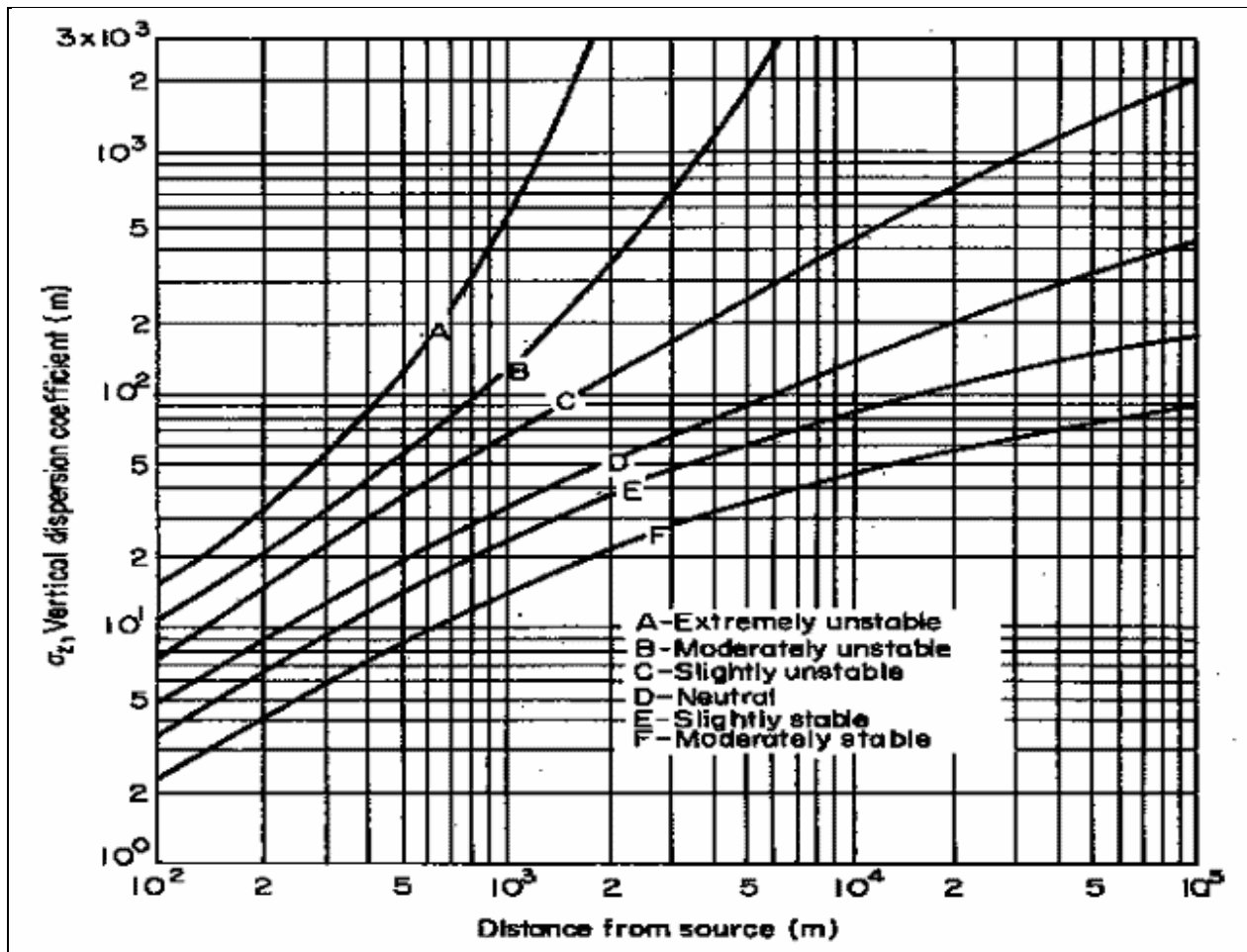


Figure 7: Vertical dispersion coefficient σ_z as a function of downwind distance from the source for various stability categories.

Figure 6 and 7 are adequate representations of the experimental results. We can use it along with **equation 17** to make predictions of concentrations downwind from the point sources. This is the most widely used method for routine calculations of air pollutant dispersion from the point source. These plots are based on measurements for $x \geq 1$. The values beyond that are based on extrapolations.

The lines are labelled A to F is correspond to different levels of atmospheric stability. On a clear and hot weather with low wind speed, the air near the ground is heated by the sun causing the air to rise and thus cause the pollutant to mix. The atmosphere is unstable thus the values of σ_y and σ_z will be larger.

On a cloudless cold night, the ground is cooled by radiation and thus cooled the air near the ground. The air is in inversion layer, making the atmosphere stable and inhibit the pollutant the values of σ_y and σ_z will be small.

Table 1: Key to stability categories

Surface wind speed (at 10 m), m/s	DAY			NIGHT	
	Incoming solar radiation			Thinly overcast or $\geq \frac{4}{8}$ cloud	Clear or $\geq \frac{3}{8}$ cloud
	Strong	Moderate	Slight		
0-2	A	A-B	B	-	-
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	D
≥ 6	C	D	D	D	D

Example:

Estimate the values of σ_y and σ_z at a point of 0.7 km downwind from a pollutant source on a bright sunny day with the wind speed 6 m/s.

(Answer: $\sigma_y = 70$ m and $\sigma_z = 45$ m)

MODIFICATIONS OF BASIC GAUSSIAN PLUME EQUATIONS

The Effect of the Ground

In last topic, the simple prediction method for the concentration in plumes at distance above the ground level had been covered. We are more interested in concentration on the ground level because there where the most people and properties are. From equation 17, it indicates that the pollutants continue to disperse at any value of z , even the z is less than 0.

The upward and downward random atmospheric motion that spread the plume in vertical direction cannot penetrate the ground. Thus, the vertical spreading terminate at $z=0$. To take this into account for the calculation, it is assumed that the pollutant is reflected upward as if the ground is a mirror. To compute the concentration that consider the ground effect, the equation is similar to the previous one accept for additional of mirror image plume that is $(z+H)^2$.

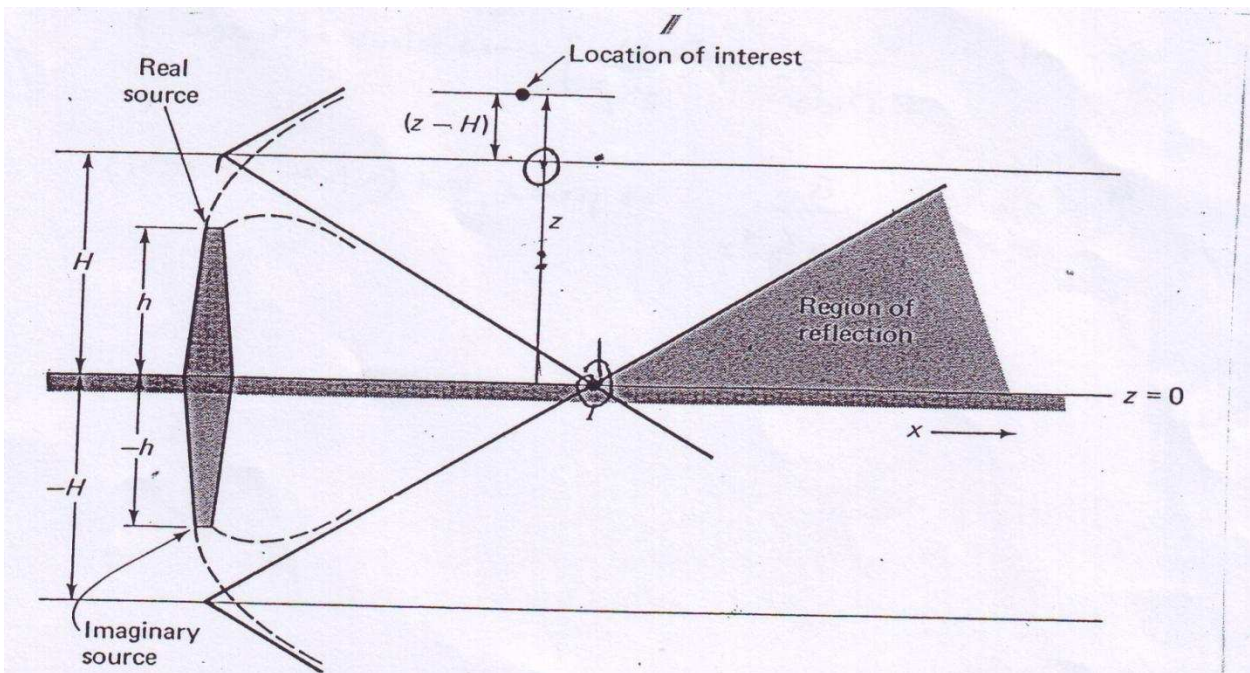


Figure 8: Use of an imaginary source to describe mathematically gaseous reflection at surface of the earth.

$$c = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp - 0.5 \left(\frac{y}{\sigma_y} \right)^2 \left[\exp - 0.5 \left(\frac{z-H}{\sigma_z} \right)^2 + \exp - 0.5 \left(\frac{z+H}{\sigma_z} \right)^2 \right] \quad \text{Equation 19}$$

When the $z = 0$, the main plume and the mirror-image plume have the same value. If the $z = H$, the main plume has a high concentration (exponent $0 = 1$), whereas for the mirror image the

value is smaller $\left[\exp - \frac{1}{2} \left(\frac{2H}{\sigma_y} \right) \right]$

Example:

A factory emits 20 g/s of CO at height (H) = 20 m. The wind speed is 5 m/s at distance 0.5 km downwind for C stability. What are the CO concentration at a point of 25m to the side and 2 m above the ground?

Solution:

$H = 20 \text{ m}; \quad Q = 20 \text{ g/s}; \quad u = 5 \text{ m/s}.$

From figure 6 and 7, read the reading for σ_y and σ_z :

$\sigma_y = 50 \text{ m}$ and $\sigma_z = 30 \text{ m}.$

Then substitute into **equation 19** to compute the concentration;

$$c = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp - 0.5 \left(\frac{y}{\sigma_y} \right)^2 \left[\exp - 0.5 \left(\frac{z-H}{\sigma_z} \right)^2 + \exp - 0.5 \left(\frac{z+H}{\sigma_z} \right)^2 \right]$$

$$c = \frac{20(\text{g/s})}{2\pi(5\text{m/s})(50\text{m})(30\text{m})} \exp - 0.5 \left(\frac{25}{50} \right)^2 \left[\exp - 0.5 \left(\frac{2-20}{30} \right)^2 + \exp - 0.5 \left(\frac{2+20}{30} \right)^2 \right]$$

$$c = 3.5 \frac{\mu\text{g}}{\text{m}^3}$$

So, the concentration at $y = 25$ m and $z = 2$ m is $3.5 \mu\text{g}/\text{m}^3$.

The equation for ground level concentration is:

$$c = \frac{Q}{\pi u \sigma_y \sigma_z} \exp -0.5 \left(\frac{y}{\sigma_y} \right)^2 \exp -0.5 \left(\frac{H}{\sigma_z} \right)^2 \quad \text{Equation 20}$$

Equation 20 is the most widely used estimating equation because it applies the greatest practical interest. For conditions of $y = 0$ and $z = 0$, which correspond to the line on the ground directly under the centre of the plume, the exponential term in y drop out of equation 19 and multiplying both side by u/Q will gives

$$\frac{cu}{Q} = \frac{1}{\pi \sigma_y \sigma_z} \exp -0.5 \left(\frac{H}{\sigma_z} \right)^2 \quad \text{Equation 21}$$

For $z = 0, y = 0$

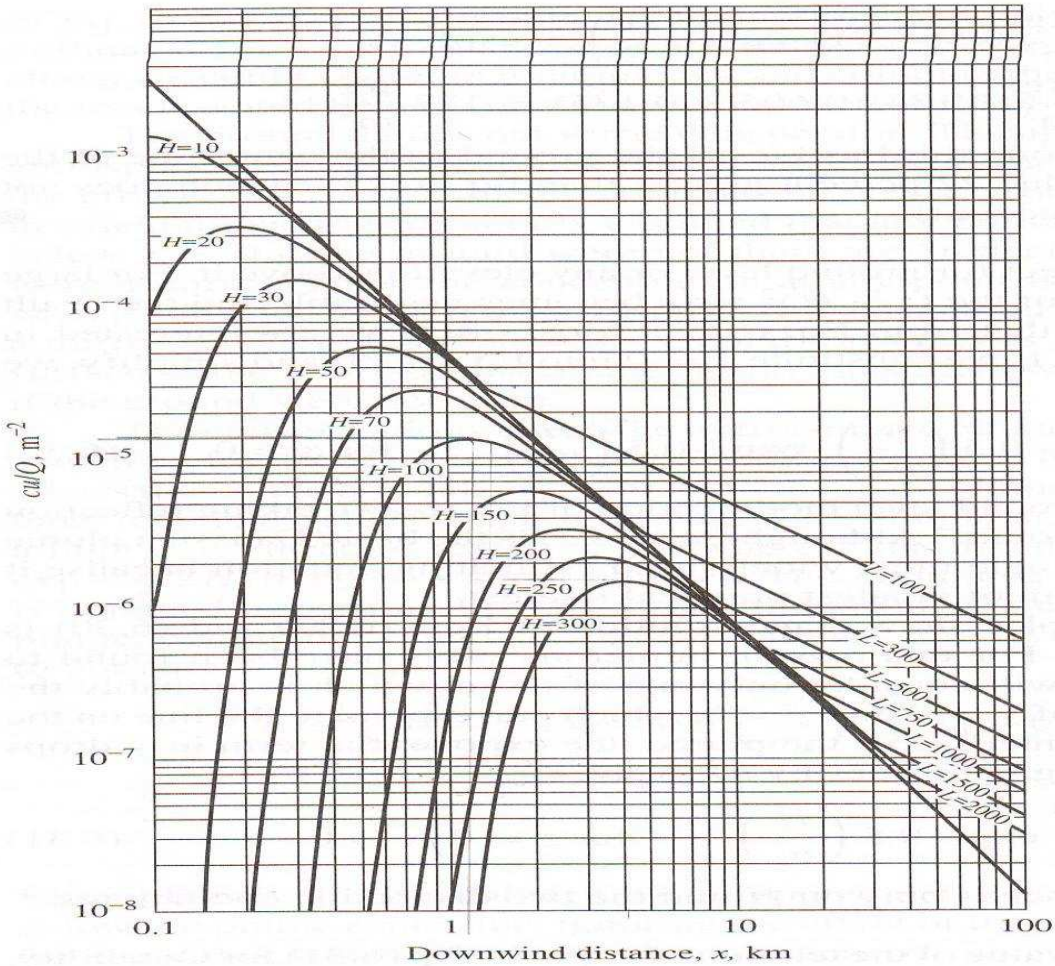


Figure 9: Ground level c_u/Q , directly under the plume centreline, as a function of downwind distance from the source and effective stack height, H , in meters, for *C stability only*.

DO IT YOURSELF:

A large, poorly copper smelter has a stack 150 m high and a plume rise of 75 m. It is currently emitting 1000 g/s of SO_2 . Estimate the ground level concentration of SO_2 from this point source at a distance 5 km directly downwind when the wind speed is 3 m/s and the stability class is C.

Answer: $638 \mu\text{g}/\text{m}^3$

DO IT YOURSELF:

A plant is emitting 750 g/s of particulates. The stack height is 100 m and the plume rise is 50 m. The wind speed is 7 m/s and the stability is C.

- What is the maximum estimated ground-level concentration?
- How far downwind does it occur?

Answer: (a) $661 \mu\text{g}/\text{m}^3$ (b) 1.9 km

PLUME RISE

Most of us have observed that the visible plumes from power plants, factories and smokestacks tend to rise and then become horizontal. Plume rise buoyantly because of; i) they are hotter than the surrounding air and; ii) they exit the stack with vertical velocity. They stop rising because as they mix with the surrounding air, they lose the velocity and cool by mixing. Finally, they level off when the temperature is the same with atmosphere.

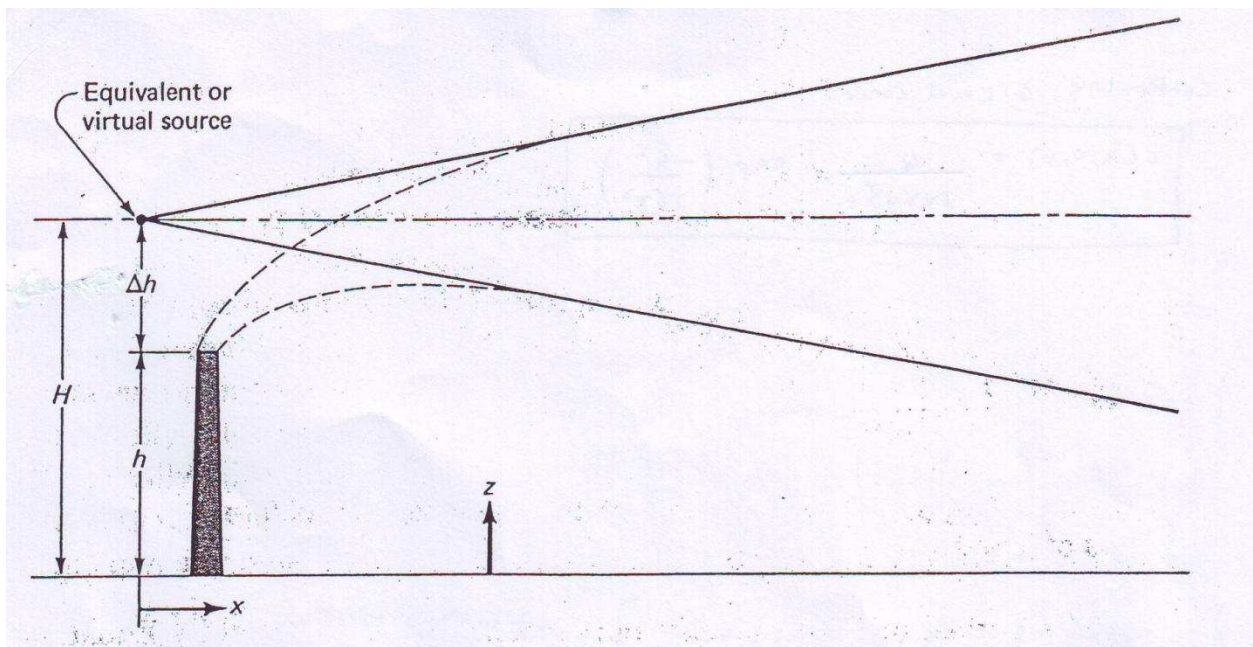


Figure10: A dispersion model with virtual source at an effective stack height H

To calculate the plume rise (Δh), this equation is applied:

$$\Delta h = \frac{V_s D}{u} \left(1.5 + 2.68 \times 10^{-3} P D \frac{(T_s - T_a)}{T_s} \right)$$

Equation 22

where;

Δh = plume rise in m

V_s = stack exit velocity in m/s

D = stack diameter in m

u = wind speed in m/s

P = pressure in millibars

T_s = stack gas temperature in K

T_a = atmospheric temperature in K

Example:

Estimate the plume rise for a 2 diameter stack whose the exit gas has a velocity of 34 m/s when the wind velocity is 4 m/s, the pressure is 1 atm and the stack and surrounding temperatures are 85 °C and 33 °C respectively.

Solution:

$$\Delta h = \frac{V_s D}{u} \left(1.5 + 2.68 \times 10^{-3} P D \frac{(T_s - T_a)}{T_s} \right)$$

$$\Delta h = \frac{(34\text{m/s})(2\text{m})}{4\text{m/s}} \left(1.5 + \frac{2.68 \times 10^{-3} \times 1013 \times 2 \times (358\text{K} - 306\text{K})}{358\text{K}} \right)$$

$$\Delta h = 38.9\text{m}$$