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قســــــــم الفيزيــــــــــاء

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Lecture (1) for Msc

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The Nature of Electromagnetism

Our physical universe is governed by four fundamental forces of nature:

- The *nuclear force* , which is the strongest of the four, but its range is limited to *subatomic scales*, such as nuclei.

- The *electromagnetic force*, which exists between all charged particles. It is the dominant force in *microscopic* systems, such as atoms and molecules, and its strength is on the order of 10^{-2} that of the nuclear force.

- The *weak-interaction force* , whose strength is only 10^{-14} that of the nuclear force. Its primary role is in interactions involving certain radioactive elementary particles.

- The *gravitational force* is the weakest of all four forces, having a strength on the order of 10^{-41} that of the nuclear force. However, it often is the dominant force in *macroscopic* systems, such as the solar system.

Electric Fields

The electromagnetic force consists of an electrical component F_e and a magnetic component F_m . The electrical force F_e is similar to the gravitational force, but with two major differences. First, *the source of the electrical field is electric charge, not mass.* Second, even though both types of fields vary inversely as the square of the distance from their respective sources, electric charges may have positive or negative polarity, resulting in a force that may be attractive or repulsive.

The fundamental quantity of charge is the quantity of a single electron, usually denoted by the letter e which has an magnitude to be given $e = 1.6 \times 10^{-19} \quad (c)$.

The charge of a single electron is q_e = -*e* and that of a proton is equal in magnitude but opposite in polarity: $q_p = +e$.

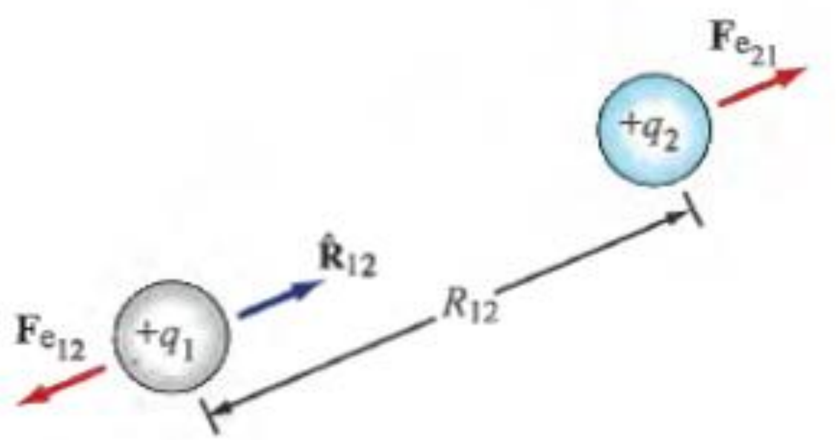


Figure (1): Electric forces on two positive point charges in free space.

Coulomb's experiments demonstrated that: *two like charges repel one another, whereas two charges of opposite polarity attract, the force acts along the line joining the charges, and its strength is proportional to the product of the magnitudes of the two charges and inversely proportional to the square of the distance between them.*

$$\boldsymbol{F_{e21}} = \widehat{\boldsymbol{R}}_{\boldsymbol{12}} \frac{\boldsymbol{q_1 q_2}}{\boldsymbol{4\pi\varepsilon_0 R_{12}^2}} \qquad (\boldsymbol{N})(\boldsymbol{in\ free\ space}) \ldots (\boldsymbol{1})$$

where F_{e21} is the *electrical force* acting on charge q_2 due to charge q_1 when both are in *free space* (vacuum), R_{12} is the distance between the two charges, $\hat{R}_{12}$ is a unit vector pointing from charge q_1 to charge q_2 (Fig.l), and *so* ε_0 is a universal constant called the *electrical permittivity of free space* [ε_0=8.854x10^{-12} farad per meter (F/m)]. The two charges are assumed to be isolated from all other charges. The force F_{e12} acting on charge q_1 due to charge q_2 is equal to force F_{e21} in magnitude, but opposite in direction: F_{e12}=- F_{e21} the *electric field intensity* **E** due to any charge q as:

$$\boldsymbol{E} = \widehat{\boldsymbol{R}} \frac{\boldsymbol{q}}{\boldsymbol{4\pi\varepsilon_0 R^2}} \qquad \left(\frac{\boldsymbol{V}}{\boldsymbol{m}}\right) \qquad \boldsymbol{infree\ space} \ldots \ldots (\boldsymbol{2})$$

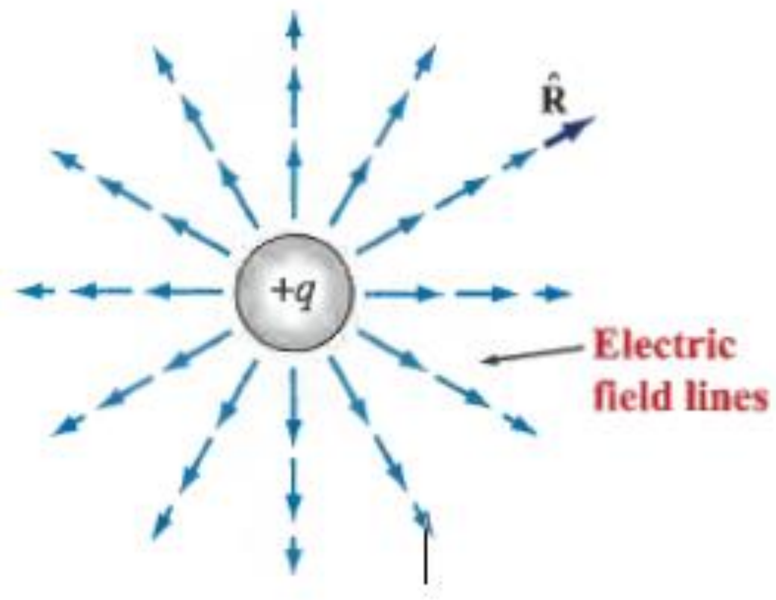


Figure (2): Electric field E due to charge *q*.

To extend Eq.(2) from the free-space case to any medium, we replace the permittivity of free space $\varepsilon_0$with ε, where ε is the permittivity of the material in which the electric field is measured and is therefore characteristic of that particular material. Thus

$$\boldsymbol{E} = \widehat{\boldsymbol{R}} \frac{\boldsymbol{q}}{\boldsymbol{4\pi\varepsilon R^2}} \qquad \left(\frac{\boldsymbol{V}}{\boldsymbol{m}}\right) \ldots .. (\boldsymbol{3})$$

Often, ε is expressed in the form:

$$\boldsymbol{\varepsilon = \varepsilon_r \varepsilon_0} \quad \text{(F/m)…..(4)}$$

Where $\boldsymbol{\varepsilon_r}$ is a dimensionless quantity called the material *relative permittivity* or *dielectric constant.* For vacuum, $\boldsymbol{\varepsilon_r}$**=1;** for air near Earth's surface, $\boldsymbol{\varepsilon_r}$ **= 1.0006.**

In addition to the electric field intensity E, we will often find it convenient to also use a related quantity called the ***electric flux density*** **D,** given by:

$$\boldsymbol{D} = \boldsymbol{\varepsilon E} \qquad .... \qquad \left(\frac{C}{m^2}\right) \ldots\ldots\ldots (5)$$

With unit of coulomb per square meter (C/m^2). These two electric quantities, E and D, constitute one of two fundamental pairs of electromagnetic fields.

Magnetic Fields

As early as 800 B.C., the Greeks discovered that certain kinds of stones exhibit a force that attracts pieces of iron. These stones are now called *magnetite* (Fe_3O_4) and the phenomenon they exhibit is known as *magnetism.* The magnetic-field pattern of a bar magnet is displayed in Fig.(3). It was also observed that like poles of different magnets repel each other and unlike poles attract each other. This attraction-repulsion property is similar to the electric force between electric charges, except for one important difference: *electric charges can be isolated, but magnetic poles always exist in pairs.*

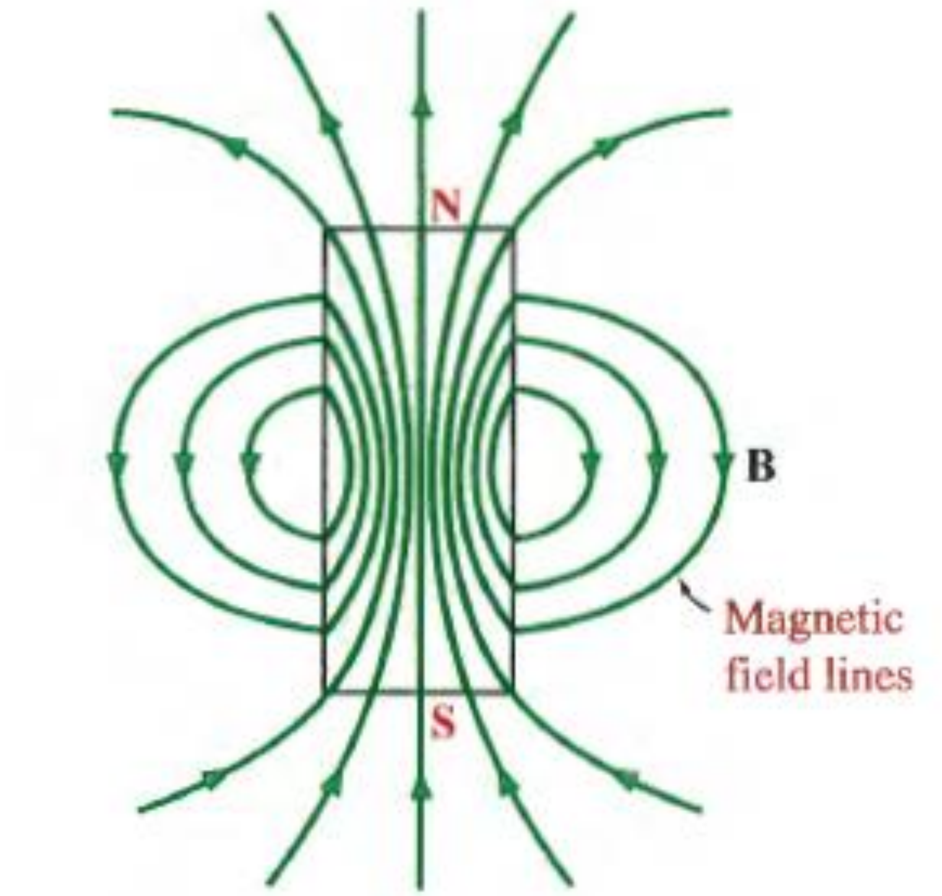


Figure (3): Pattern of magnetic field lines around a bar magnet.

The magnetic lines surrounding a magnet represent the *magnetic flux density* B. A magnetic field not only exists around permanent magnets but can also be created by electric current. The current-carrying wire induced a magnetic field that formed closed circular loops around the wire Fig.(4).

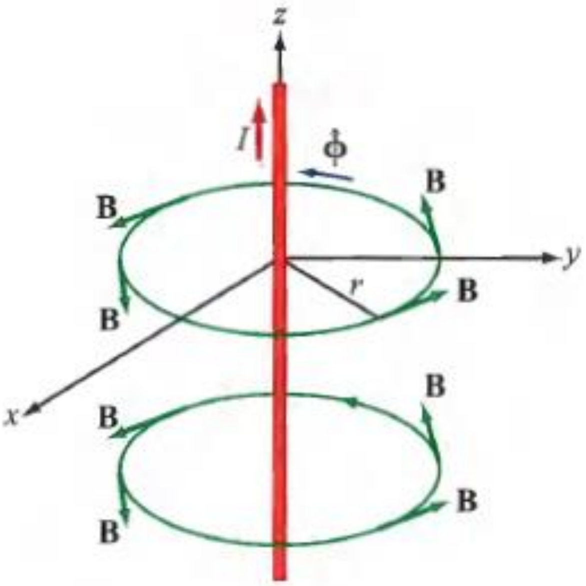


Figure (4) : The magnetic field induced by a steady current flowing in the z- direction

The ***magnetic flux density*** **B** induced by a constant current ***I*** flowing in the z-direction is given by:

$$\boldsymbol{B} = \hat{\boldsymbol{\varphi}} \frac{\boldsymbol{\mu_0 I}}{\boldsymbol{2\pi r}} \qquad (\boldsymbol{T}) \ldots . (\boldsymbol{6})$$

Where ***r*** is the radial distance from the current and $\hat{\boldsymbol{\varphi}}$ is an azimuthal unit vector expressing the fact that the magnetic field direction is tangential to the circle surrounding the current. The magnetic field is measured in tesla (T), named. The quantity $\mu_0$is called the *magnetic permeability of free s*pace [$\boldsymbol{\mu_0=4\pi x 10^{-7}}$ **henry per meter (H/m)**]. The product of ε_0 and μ_0 *according to the following equation* specifies *c,* the *velocity of light in free space:*

$$\boldsymbol{c} = \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{\varepsilon_0 \mu_0}}} = \boldsymbol{3 \times 10^8} \qquad .. \left(\frac{m}{s}\right) \ldots . (7)$$

The electric force on charge q is **F_e=qE** and the Magnetic force on moving charge q by velocity $\boldsymbol{v}$ is **F_m=q**$\boldsymbol{v} \times \boldsymbol{B}$

To extend Eq.(6) to a medium other than free space, $\boldsymbol{\mu_0}$ should be replaced with $\boldsymbol{\mu}$, the *magnetic permeability* of the material in which **B** is being observed. The majority of natural materials are *nonmagnetic,* meaning that they exhibit a magnetic permeability $\boldsymbol{\mu=\mu_0}$. *While* For ferromagnetic materials, such as iron and nickel, $\boldsymbol{\mu}$ can be much larger than μ_0 The magnetic permeability $\boldsymbol{\mu}$ accounts for *magnetization* properties of a material. In analogy with Eq.(4), $\boldsymbol{\mu}$ of a particular material can be defined as:

$$\mu = \mu_r \, \mu_0 \qquad \text{(H/m)}, \ldots..(8)$$

where μ_r is a dimensionless quantity called the *relative magnetic permeability* of the material.

We stated earlier that **E and D** constitute one of two pairs of electromagnetic field quantities.

$$D = \varepsilon E \quad \ldots. \quad \left(\frac{C}{m^2}\right) \ldots\ldots\ldots (9)$$

The second pair is **B** and the *magnetic field intensity* **H**, which are related to each other through *μ:*

$$B = \mu H \ldots\ldots \quad (T) \ldots (10)$$

Table : The three branches of electromagnetics.

Branch	Condition	Field Quantities (Units)
Electrostatics	Stationary charges $(\partial q/\partial t = 0)$	Electric field intensity **E** (V/m) Electric flux density **D** (C/m^2) $\mathbf{D} = \varepsilon \mathbf{E}$
Magnetostatics	Steady currents $(\partial I/\partial t = 0)$	Magnetic flux density **B** (T) Magnetic field intensity **H** (A/m) $\mathbf{B} = \mu \mathbf{H}$
Dynamics (Time-varying fields)	Time-varying currents $(\partial I/\partial t \neq 0)$	**E**, **D**, **B**, and **H** (**E**, **D**) coupled to (**B**, **H**)

Table : Constitutive parameters of materials.

Parameter	Units	Free-space Value
Electrical permittivity ε	F/m	$\varepsilon_0 = 8.854 \times 10^{-12}$ (F/m) $\simeq \frac{1}{36\pi} \times 10^{-9}$ (F/m)
Magnetic permeability μ	H/m	$\mu_0 = 4\pi \times 10^{-7}$ (H/m)
Conductivity σ	S/m	0

Traveling Waves:

Waves are a natural consequence of many physical processes: waves manifest themselves as ripples on the surfaces of oceans and lakes; sound waves constitute pressure disturbances that travel through air; mechanical waves modulate stretched strings; and electromagnetic waves carry electric and magnetic fields through free space and material media as microwaves, light, and X-rays. All these various types of waves exhibit a number of common properties, including:

Moving waves carry energy.

• *Waves have velocity.* It takes time for a wave to travel from one point to another. Electromagnetic waves in vacuum travel at a speed of $3x10^8$ m/s, and sound waves in air travel at a speed approximately a million times slower, specifically 330m/s.

• *Many waves exhibit a property called linearity.* Waves that do not affect the passage of other waves are called *linear* because they can pass right through each other. The total of two linear waves is simply the sum of the two waves as they would exist separately. Electromagnetic waves are linear, as are sound waves.

Waves are of two types: ***transient waves*** caused by sudden disturbances and ***continuous periodic*** *waves* generated by a repetitive source.

An essential feature of a propagating wave is that it is a self-sustaining disturbance of the medium through which it travels. If this disturbance varies as a function of one space variable, such as the vertical displacement of the string shown in **Fig(5)**, we call the wave *one-dimensional*. The vertical displacement varies with time and with the location along the length of the string. Even though the string rises up into a second dimension, the wave is only one dimensional because the disturbance varies with only one space variable.

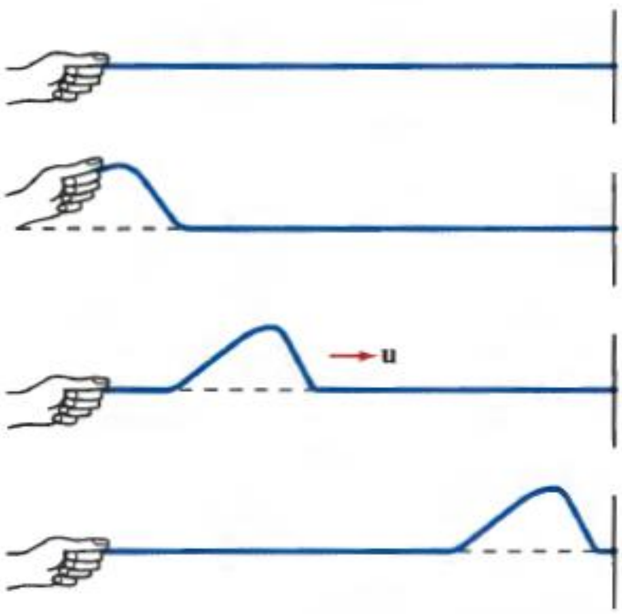


Figure (5): A one-dimensional wave traveling on a string.

Circular, cylindrical and spherical waves see fig(6).

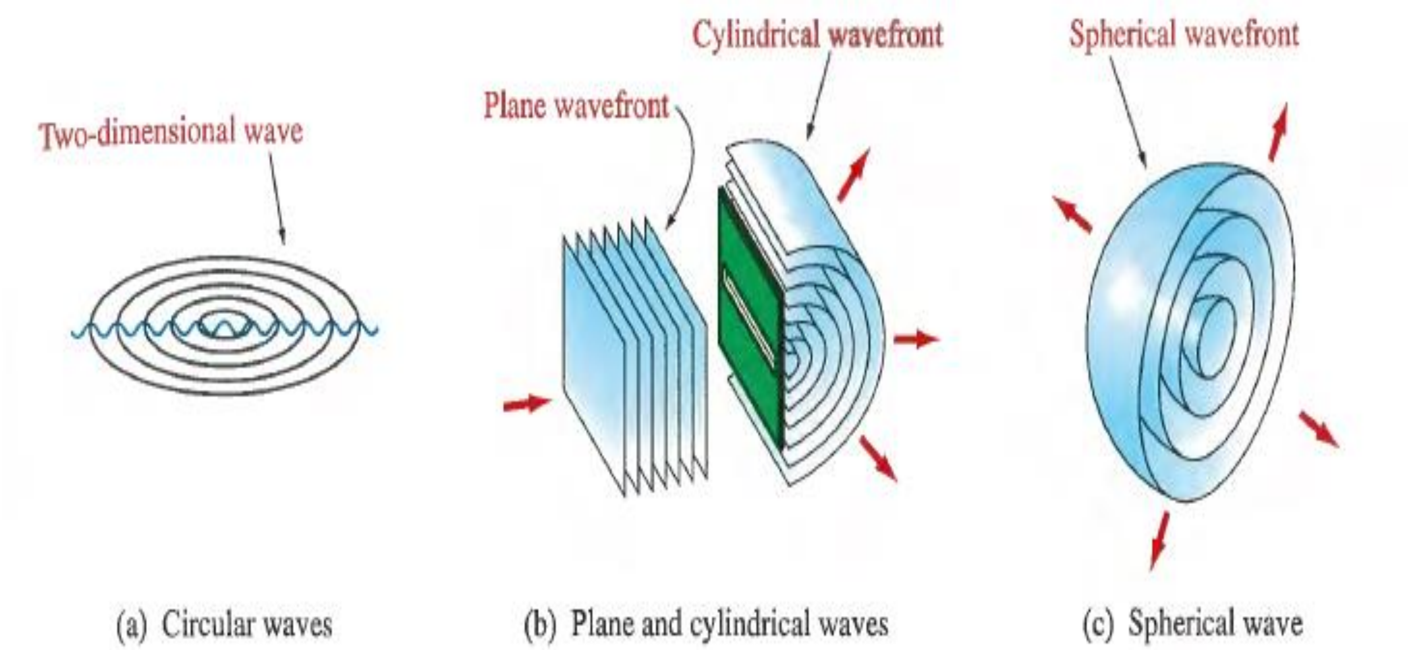


Figure (6): Examples of two-dimensional and three-dimensional waves: (a) circular waves on a pond, (b) a plane light wave exciting a cylindrical light wave through the use of a long narrow slit in an opaque screen, and (c) a sliced section of a spherical wave.

Wave equation:

The general formula of differential wave equation given by:

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \quad \ldots\ldots(1)$$

Or

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \ldots\ldots(2)$$

Where $y\,(x,t)$ wave equation for wave travel in speed v. Use partial differential equation to solve eq.(1) or (2):

Suppose :

$$y = Q(x)R(t) \quad \ldots\ldots(3)$$

So eq.(1) becomes:

$$\frac{1}{R}\frac{\partial^2 R}{\partial t^2} = \frac{v^2}{Q}\frac{\partial^2 Q}{\partial x^2} = A = separation\ constant \quad \ldots\ldots(4)$$

So get:

$$\frac{1}{R}\frac{\partial^2 R}{\partial t^2} = A$$

$$\frac{\partial^2 R}{\partial t^2} = AR(t) \text{.......................(5)}$$

$$\frac{\partial^2 Q}{\partial x^2} = \frac{A}{v^2} Q(x) \text{ ....(6)}$$

Now need to solve eq.(5) & (6):

Case (1): when A=0

$\therefore$

$$\frac{\partial^2 R}{\partial t^2} = 0$$

$$\frac{\partial R}{\partial t} = a = constant$$

$$R = at + b \text{...............(7)}$$

&

$$\frac{\partial^2 Q}{\partial x^2} = 0$$

$$\frac{\partial Q}{\partial x} = c = constant$$

$$Q = Cx + d \text{...............(8)}$$

When a,b,c &d are constant. $\therefore$ $y = Q.R$ = **(at+b)(cx+d)**

$$y = a_1 + b_1 x + c_1 t + d_1 xt \text{ ....(9)}$$

Case (2): when **A** = $\omega^2 > 0$ get:

$$\frac{\partial^2 R}{\partial t^2} = AR(t) = \omega^2\, R(t) \text{ ....(10)}$$

&

$$\frac{\partial^2 Q}{\partial x^2} = (\frac{\omega}{v})^2 Q \text{ ....(11)}$$

H.W (1): find $y(x, t)$.

Case (3): when $\mathbf{A = -\omega^2 < 0}$ **:** H.W(2) find $y(x,t)$.

H.W (3) : Solve the following wave eq.

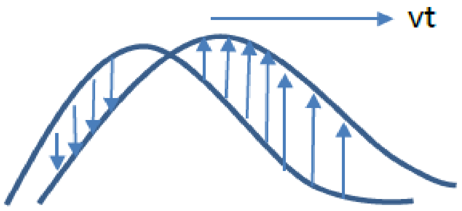
$$9\,\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2} \qquad 0 < x < \pi$$

$$t > 0$$

$$y(0,t)=0\,, \qquad y(\pi,t)=0 \qquad t > 0$$

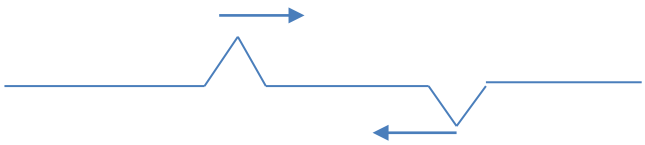
$$y(x,0)=\mathrm{f(x)}\,, \qquad y(x,0)=0 \quad 0 \le x \le \pi$$

$$\mathrm{f}(\mathrm{x}) = \begin{cases} \mathrm{x} & 0 \le x \le \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \le x \le \pi \end{cases}$$



What is moving?

Wave equations are linear: this means that a linear combination of solution is a solution.

1-


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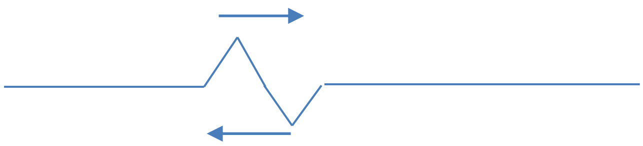

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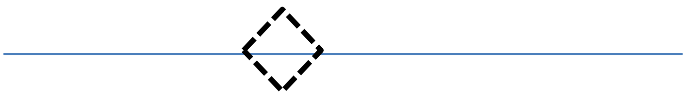
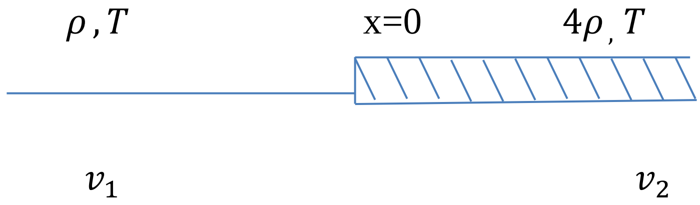


This case different from stationary string: the energy stored in the string.

4-

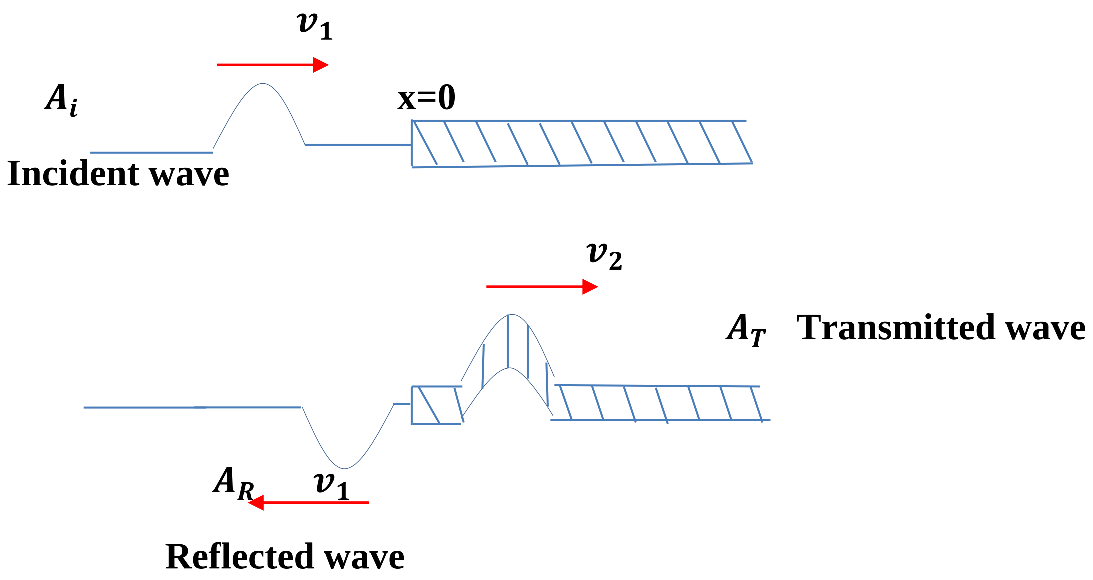

If we have a string with different thicknesses (i.e. different densities (ρ) & tensions (T)):



Assuming that the tension (T) is uniform:

$$v_1 = \sqrt{\frac{T}{\rho}} \quad \& \quad v_2 = \sqrt{\frac{T}{4\rho}} = \frac{1}{2} v_1$$

The velocity of a wave v_2 in a denser string$(4\rho)$for example, is slower than $v_1$by half. The wave will move through this string as:



When a wave moves from one medium to another, the wavelength will be changed but the frequency will be constant.

$$Rf = \frac{v_2 - v_1}{v_2 + v_1} \qquad \textbf{Reflected (Rf)}$$

$$\text{Tr}= \frac{2v_2}{v_2 + v_1} \qquad \textbf{Transmitted (Tr)}$$

$$v_2 = \frac{v_1}{2},$$

$$\therefore \qquad \boldsymbol{Rf = -\frac{1}{3}} \text{ , the phase change by } (\pi);$$

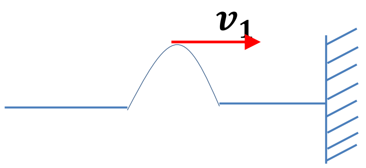
$$\boldsymbol{Tr = \frac{2}{3}} \text{ , no phase change.}$$

Impedance: $\boldsymbol{Z_1 = \frac{T}{v_1}}$ **&** $\boldsymbol{Z_2 = \frac{T}{v_2}}$

The amplitude of the transmitted and reflected wave is determined by the properties of the two media (systems).

Consider two extreme cases:

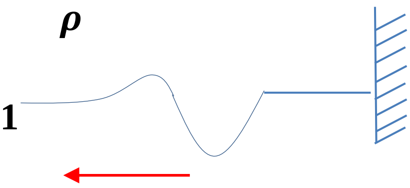
1. String attached a wall:



Wall

$\boldsymbol{v_2}$**=0**

Rf=-1

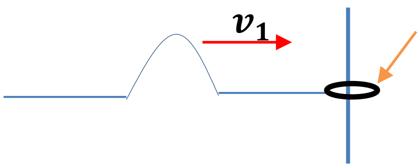


$\boldsymbol{\rho}$ **= big**

Tr=0

In a sense, the (ρ) of the wall is very big, infinite in fact. Therefore, ($\boldsymbol{v_2 \approx 0}$ **Rf= -1 & Tr= 0).** The amplitude changes sign but not magnitude, and there is no transmitted wave.

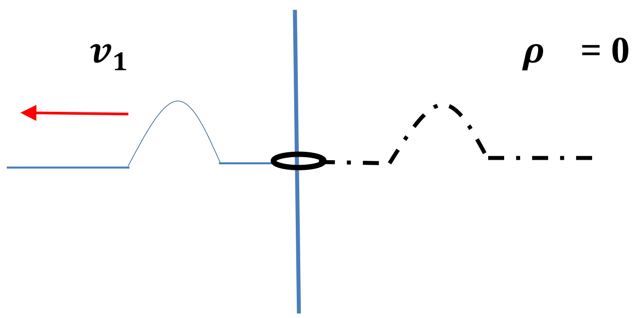
2. There is air on the other side: the (ρ) of the air is zero, therefore, ($\boldsymbol{v_2 = \infty}$**, Rf=1 & Tr=2).**



Massless ring

Air

$\boldsymbol{v_{2=\infty}}$



Sinusoidal Waves in a Lossless Medium

Regardless of the mechanism responsible for generating them, all linear waves can be described mathematically in common terms. *A medium is said to be loss less if it does not attenuate the amplitude of the wave traveling within it or on its surface.* By way of an example, let us consider a wave traveling on a lake surface, and let us assume for the time being that frictional forces can be ignored, thereby allowing a wave generated on the water surface to travel indefinitely with no loss in energy.

If *y* denotes the height of the water surface relative to the mean height (undisturbed condition) and *x* denotes the distance of wave travel, the functional dependence of *y* on time *t* and the spatial coordinate *x* has the general form:

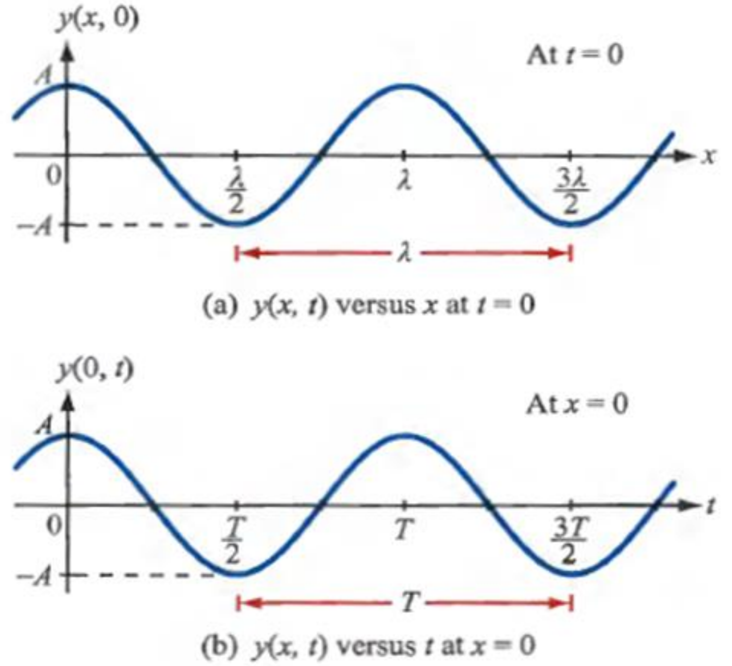
$$\boldsymbol{y(x,t) = A\,cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \varphi_0\right)} \quad .... \quad (\boldsymbol{m}) \ldots (\mathbf{1})$$

Where ***A*** is the *amplitude* of the wave, ***T*** is its *time period*, **λ** is its *spatial wavelength*, and $\boldsymbol{\varphi_o}$ is a *reference phase.* The quantity *y(x, t)* can also be expressed in the form

$$\boldsymbol{y(x,t) = A\,cos\big(\varphi(x,t)\big)} \qquad \ldots (\boldsymbol{m})\boldsymbol{....}$$

Where

$$\boldsymbol{\varphi(x,t)} = \left(\boldsymbol{\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \varphi_0}\right) \qquad (\boldsymbol{rad}) \qquad (\mathbf{2})$$



figure(1) Plots of $y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$ as a function of (a) x at $t = 0$ and (b) t at $x = 0$.

The angle $\boldsymbol{\varphi(x,t)}$ is called the *phase* of the wave, and it should not be confused with the reference phase $\boldsymbol{\varphi_0}$, which is constant with respect to both time and space. Phase is measured by the same units as angles, that is, radians (rad) or degrees, with $\boldsymbol{2\pi}$ radians=360°.

Let us first analyze the simple case when $\boldsymbol{\varphi_0 = 0}$:

$$\boldsymbol{y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) \quad .... \quad (m) \ldots (3)}$$

The plots in Fig.(1) show the variation of ***y(x, t)*** with ***x*** **at t = 0** and with ***t*** **at *x*=0.** The wave pattern repeats itself at a spatial period $\boldsymbol{\lambda}$ along ***x*** and at a temporal period ***T*** along ***t***. We can measure the *phase velocity* of the wave. At the peaks of the wave pattern, the phase $\varphi(x,t)$ is equal to zero or multiples of 2π radians. Thus;

$$\boldsymbol{\varphi(x,t) = \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} = 2n\pi, \quad n = 0,1,2 \ldots. \qquad \ldots (4)}$$

i.e phase $\varphi(x,t)$ is constant, by taking the time derivative of Eq. (4)get:

$$\boldsymbol{\frac{2\pi}{T} - \frac{2\pi}{\lambda}\frac{dx}{dt} = 0, \ldots \ldots .. (5),}$$

Which gives the *phase velocity* $\mathbf{u_p}$ as:

$$\boldsymbol{u_p = \frac{dx}{dt} = \frac{\lambda}{T}, \qquad .. \left(\frac{m}{s}\right) \ldots (6)}$$

The phase velocity, also called the *propagation velocity*, is *the velocity of the wave pattern* as it moves across the water surface. The water itself mostly moves up and down; when the wave moves from one point to another, the water does not move physically along with it. The *frequency* of a sinusoidal wave, *f*, is the reciprocal of its time period T:

$$\boldsymbol{f = \frac{1}{T}, \qquad .. \ (Hz) \ldots (7)}$$

Combining the preceding two equations yields:

$$\boldsymbol{u_p = f\lambda, \qquad .. \left(\frac{m}{s}\right) \ldots (8)}$$

The wave frequency *f*, which is measured in cycles per second, has been assigned the unit (Hz). Using Eq. (8), Eq. (3) can be rewritten in a more compact form as:

$$\boldsymbol{y(x,t) = A\cos\left(2\pi f t - \frac{2\pi}{\lambda}x\right) = A\cos(\omega t - \beta x) \quad \ldots (9)}$$

Where $\boldsymbol{\omega}$ is the *angular velocity* of the wave and $\boldsymbol{\beta}$ is its *phase constant* (or *wavenumber)*, defined as:

$$\boldsymbol{\omega = 2\pi f, \qquad \left(\frac{rad}{s}\right), \ \beta = \frac{2\pi}{\lambda}, \qquad \left(\frac{rad}{m}\right) \quad ..(10)}$$

In terms of these two quantities,

$$u_p = f\lambda = \frac{\omega}{\beta} \quad \ldots\ldots..(11)$$

So far, we have examined the behavior of a wave traveling in the +x-direction. To describe a wave traveling in the -x-direction, we reverse the sign of x in Eq.(9):

$$y(x,t) = A\cos(\omega t + \beta x) \quad \ldots(12)$$

We now examine the role of the phase reference φ_0 given previously in Eq.(1). If φ_0 is not zero, then Eq. (9) should be written as:

$$y(x,t) = A\cos\left(\omega t - \beta x + \varphi_0\right) \quad \ldots(13)$$

A plot of *y(x,* t) as a function of *x* at a specified *t* or as a function of *t* at a specified *x* will be shifted in space or time, respectively, relative to a plot with $\varphi_0 = 0$ by an amount proportional to φ_0. This is illustrated by the plots shown in Fig.(2). We observe that when φ_0 is positive, *y (t)* reaches its peak value, or any other specified value, sooner than when $\varphi_0 = 0$. Thus, the wave with φ_0=π/4 is said to *lead* the wave with $\varphi_0 = 0$ by a *phase lead* of π /4; and similarly, the wave with φ_0= -π/4 is said to *lag* the wave with φ_0=0 by a *phase lag* of π/4. A wave function with a negative φ_0 takes longer to reach a given value of *y(t),* such as its peak, than the zero-phase reference function.

When its value is positive, φ_0 signifies a phase lead in time, and when it is negative, it signifies a phase lag.

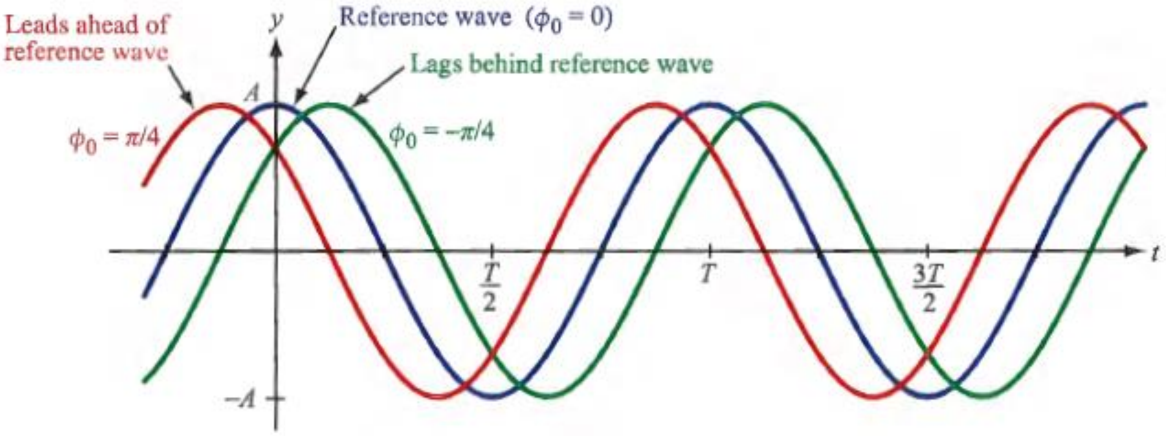
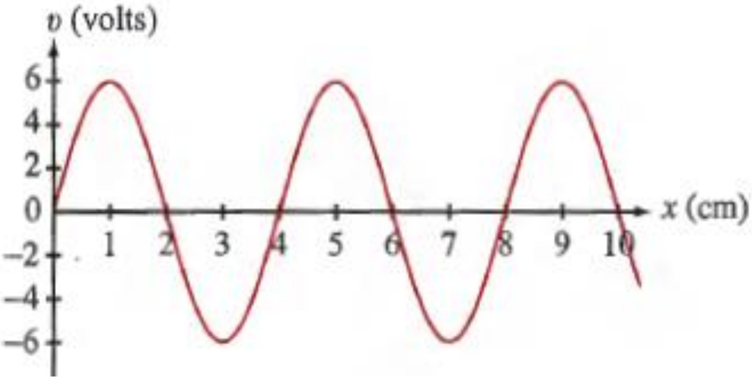


Figure (2): Plots of *y(0, t)* = *A* cos [(2 π*t/T)*+ φ_0] for three different values of the reference phase φ_0.

Example: Consider the red wave shown in Fig.(3). What is the wave's
(a) amplitude, (b) wavelength, and (c) frequency, given that its phase velocity is 6 m/s?



Fig(3)

Solution:

a) A = 6 V, b) λ = 4 cm., c) $f \frac{u_p}{\lambda} = \frac{6}{4\times10^{-2}} = \mathbf{150}\ \boldsymbol{Hz}$

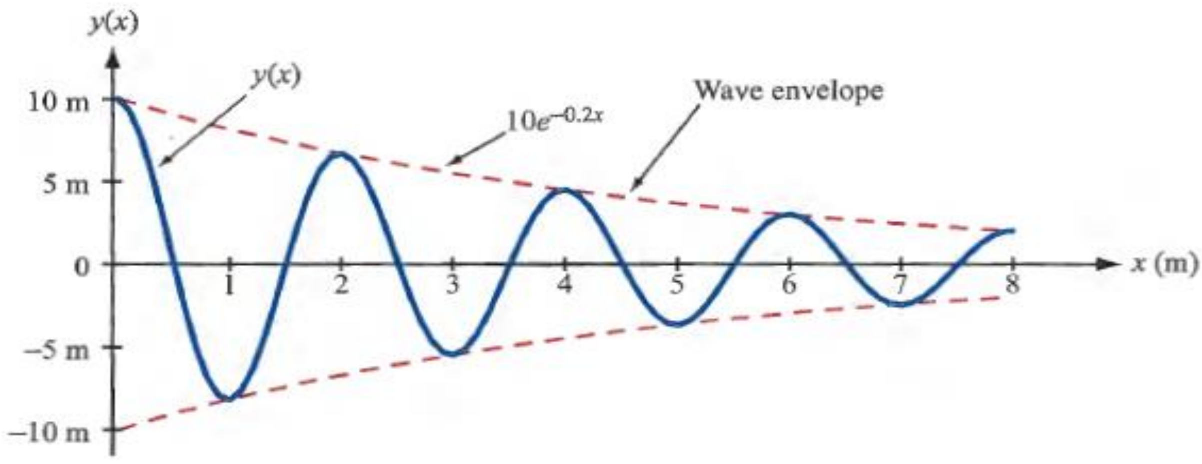
Sinusoidal Waves in a Lossy Medium

If a wave is traveling in the x-direction in a *lossy medium,* its amplitude will decrease as $\boldsymbol{e^{-\alpha x}}$. This factor is called the *attenuation factor*, and $\boldsymbol{\alpha}$ is called the *attenuation constant* of the medium and its unit is neper per meter (Np/m). Thus, in general:

$$\boldsymbol{y(x,t) = A\, e^{-\alpha x} \cos\left(\omega t - \beta x + \varphi_0\right)} \quad \dots \mathbf{(13)}$$

The wave amplitude is now $\boldsymbol{A\, e^{-\alpha x}}$, and not just $\boldsymbol{A}$. Figure (4) shows a plot of *y(x, t)* as a function of *x* at t = 0 for $\boldsymbol{A}$ = 10m, $\boldsymbol{\lambda}$ = 2 m, $\boldsymbol{\alpha}$ = 0.2 Np/m, and φ_0=0. Note that the envelope of the wave pattern decreases as $\boldsymbol{e^{-\alpha x}}$**.**

The real unit of $\boldsymbol{\alpha}$ **is (m^{-1}); the neper (Np)** part is a dimensionless, artificial adjective traditionally used as a reminder that the unit (Np/m) refers to the attenuation constant of the medium, $\boldsymbol{\alpha}$, A similar practice is applied to the phase constant $\boldsymbol{\beta}$ by assigning it the unit (**rad/m**) instead of just (**m^{-1}**).



Fig(4): Plot of $\boldsymbol{y(x) = (10e^{-0.2x} \cos(\pi x))}$meters. Note that the envelope is bounded between the curve given by $\mathbf{10e^{-0.2x}}$ and its mirror image.

Example: An acoustic wave traveling in the x-direction in a fluid (liquid or gas) is characterized by a differential pressure p(x,t). The unit for pressure is newton per square meter (N/m^2).

Find an expression for p(x,t) for a sinusoidal sound wave traveling in the positive x-direction in water, given that the wave frequency is **1** kHz, the velocity of sound in water is 1.5 km/s, the wave amplitude is 10 N/m2 , and p(x,t) was observed to be at its maximum value at *t* = 0 and *x* = 0.25 m. Treat water as a lossless medium.

Solution: According to the general form given by Eq.(1) for a wave traveling in the positive x-direction,

$$p(x,t) = A\cos\left(2\pi f t - \frac{2\pi}{\lambda}x + \varphi_0\right) \dots (N/m^2)$$

The amplitude A= **10** N/m^2 , $T = 1/f = 10^{-3}$ s, and from $u_p = f\lambda$

$$\lambda = \frac{u_p}{f} = \frac{1.5\times10^3}{10^3} = 1.5\text{m}$$

Hence

$$p(x,t) = 10\cos\left(2\pi\, 10^3 t - \frac{4\pi}{3}x + \varphi_0\right) \dots (N/m^2)$$

Since at $t = 0$ **and** $x = 0.25$ **m, p(0.25,0)=10N/m**^{2}, we have

$$10 = 10\cos\left(\frac{-4\pi}{3}0.25 + \varphi_0\right) = 10\cos\left(\frac{-\pi}{3} + \varphi_0\right)$$

$$\cos\left(\varphi_0 - \frac{\pi}{3}\right) = 1$$

$$\varphi_0 - \frac{\pi}{3} = cos^{-1}(1) = 0$$

$$\varphi_0 = \frac{\pi}{3}$$

which yields the result $\varphi_0 = \pi/3$.

Hence,

$$p(x,t) = 10\cos\left(2\pi\, 10^3 t - \frac{4\pi}{3}x + \frac{\pi}{3}\right) \dots (N/m^2)$$

Example : Power Loss: A laser beam of light propagating through the atmosphere is characterized by an electric field given by:

$$E(x,t) = 150e^{-0.03x}\cos(3\times10^{15}t - 10^7 x) \quad \dots \left(\frac{V}{m}\right)$$

where *x* is the distance from the source in meters. The attenuation is due to absorption by atmospheric gases. Determine:

a) the direction of wave travel,

b) the wave velocity, and

c) the wave amplitude at a distance of 200 m.

Solution:

a) Since the coefficients of t and *x* in the argument of the cosine function have opposite signs, the wave must be traveling in the +x-direction.

b) $$u_p = \frac{\omega}{\beta} = \frac{3\times10^{15}}{10^7} = 3\times10^8 \quad m/s$$

This is equal to **c**, the velocity of light in free space.

(c) *At* ***x*** **=200**m, the amplitude of ***E(x, t)*** is

$$E(200, t) = 150 \times e^{-0.03\times200} \quad = 0.37 \quad (V/m)$$

References

1- Fawwaz T. Ulaby,Eric Michielssen,Umberto Ravaioli'sFundamentals of Applied Electromagnetics (6th Edition) [Hardcover], Prentice Hall**,** 2010.

2- Jackson, John D. (1998). *Classical Electrodynamics (3rd ed.)*. Wiley