# 13. Fundamental Forces

## 13.1 Forces for Vertical Air Motions

Acting forces:
- vertical pressure gradient force per unit mass: $\boldsymbol{a}_{p,v} = -1/\rho\ \partial p/\partial z$
- gravitational force: $\boldsymbol{a}_g = g$

Equation of motion:

$$\underbrace{\frac{dv_v}{dt}}_{10^{-7}\ ms^{-2}} = -\underbrace{\frac{1}{\rho}\frac{\partial p}{\partial z}}_{10\ ms^{-2}} - \underbrace{g}_{10\ ms^{-2}} \tag{13.1}$$

If typical magnitudes for the accelerations given in Eq. (13.1) are assessed, it is shown that in most cases in good approximation the vertical acceleration can be neclected compared to the driving forces in the vertical direction. Thus, the vertical pressure gradient force and the gravitational force are balanced (in large scale). This state is called the *hydrostatic equilibrium*:

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} - g = 0 \tag{13.2}$$

or

$$\frac{\partial p}{\partial z} = -g\rho, \tag{13.3}$$

where $g$ = 9.81 ms$^{-2}$ is the gravitational acceleration at sea level.

For an atmosphere in a hydrostatic balance the decrease in pressure near the ground ($\rho = \rho_0$) can be calculated from Eq. (13.3):

$$\partial p/\partial z = -1.225\ \text{kg m}^{-3} \cdot 9.81\ \text{ms}^{-2} = -0.12\ \text{hPa/m} \tag{13.4}$$

and

$$\partial z/\partial p = -8.3\ \text{m/hPa}. \tag{13.5}$$

## 13.2 Forces for Horizontal Air Motion

Fluid elements are influenced by
– horizontal pressure gradient force
– Coriolis force
– friction (viscous) force

### 13.2.1 Horizontal Pressure Gradient Force

From spatial pressure differences given by the horizontal pressure gradient $\boldsymbol{\nabla}_{\boldsymbol{h}} p$ it follows:

$$\boldsymbol{F}_{h,p} = -\frac{m}{\rho}\boldsymbol{\nabla}_{\boldsymbol{h}} p \tag{13.6}$$

and

$$\boldsymbol{a}_{h,p} = -\frac{1}{\rho}\boldsymbol{\nabla}_{\boldsymbol{h}} p. \tag{13.7}$$

$\boldsymbol{F}_{h,p}$ and $\boldsymbol{a}_{h,p}$ are directed perpendicular to the isobars towards low pressure.

### 13.2.2 Coriolis force

Particle movements on the earth can be separated into two parts:
– movement with respect to the earth surface
– rotation with the earth.

In a rotating coordinate system (fixed to the earth's surface) an additional virtual [1] force has to be considered, the *Coriolis force*. It acts only on moving particles. (This is different from the centrifugal force, an other virtual force that acts even on particles in rest with respect to the earth surface.)

Qualitative description
Example: Meridional movement of a particle in North-South direction.
The rotational speed of a particle depends on latitude and decreases from its maximum value at the equator (1670 km/h) to 0 km/h at the poles.

For particles moving from the equator towards North the principle of conservation of angular momentum causes a higher rotational speed as that of their environment. For an observer on the earth surface (i.e. on the rotational reference system) this is an virtual deflection of the particle to the right.
(Note: This is valid for motions in the northern hemisphere. In the southern hemisphere the deflection is to the left.)

Example: Zonal movements in East-West direction.
Motions in zonal directions lead to changes in the centrifugal force $\Omega^2 R$: eastward: increasing , westward: decreasing.

[1] This force is of a virtual nature because of its abscence in an absolute space-fixed coordinate system.

Motions from West to East cause an acceleration towards the equator and vertically upward (East to West vice versa). This again is a virtual deflection to the right (in the northern hemisphere).

Mathematical description:

A formal description of the apparent forces acting on moving objects in a rotating system can be derived by a transformation of coordinates between the inertial reference frame and the reference frame rotating with the angular velocity of the earth. The motion of an object with respect to a rotating system can be described by the relationship between the total derivatives [2] of a position vector in an inertial reference frame and in the rotating system:

$$\frac{d'\boldsymbol{r}}{dt} = \frac{d\boldsymbol{r}}{dt} - [\boldsymbol{r} \times \boldsymbol{\Omega}], \tag{13.8}$$

where $d'\boldsymbol{r}/dt$ and $d\boldsymbol{r}/dt$ are the respective rates of change of $\boldsymbol{r}$ in the inertial and the rotating system and the second term on the right-hand side is the contribution due to the rotation of the system with the angular velocity $\Omega$. Applying Eq. (13.8) to the velocity vector $\boldsymbol{v} = d'\boldsymbol{r}/dt$ gives

$$\frac{d'}{dt}\frac{d'\boldsymbol{r}}{dt} = \frac{d}{dt}\frac{d'\boldsymbol{r}}{dt} - \left[\frac{d'\boldsymbol{r}}{dt} \times \boldsymbol{\Omega}\right] \tag{13.9}$$

[2] The total derivative is the rate of change of a given quantity following the motion.

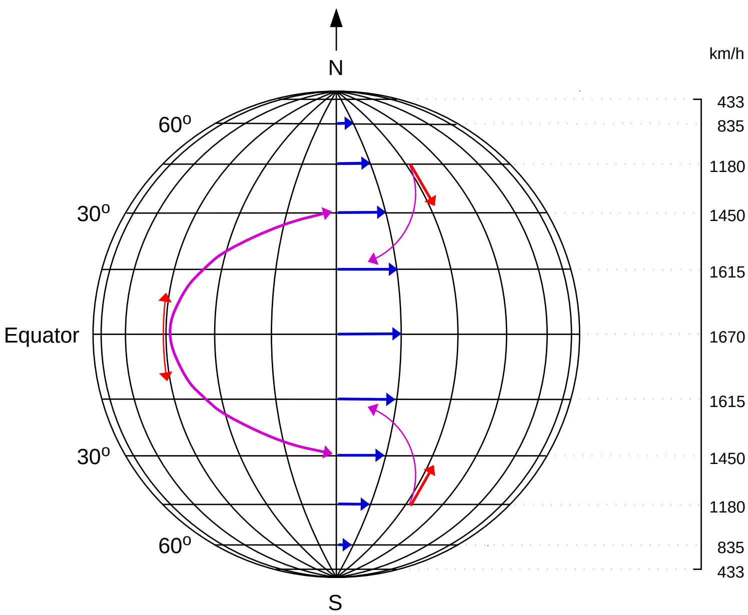


**Fig. 13.1.** The Coriolis force acting on particles which move meridionally. The right scale gives the rotational speed of the earth surface for a given latitude.