The relation between $\lambda, a_2, C_{P,max}$ is given in Table 1.

λ	a_2	$C_{P,max}$
0.5	0.2983	0.289
1.0	0.3170	0.416
1.5	0.3245	0.477
2.0	0.3279	0.511
2.5	0.3297	0.533
5.0	0.3324	0.570
7.5	0.3329	0.581
10	0.3330	0.585

Table 1

The numerical results of (43) are shown in Fig. 5, and induction factors: axial (a), rotational (a') are plotted as a function of $\left(\frac{r}{R}\right)$ in Fig. 6.

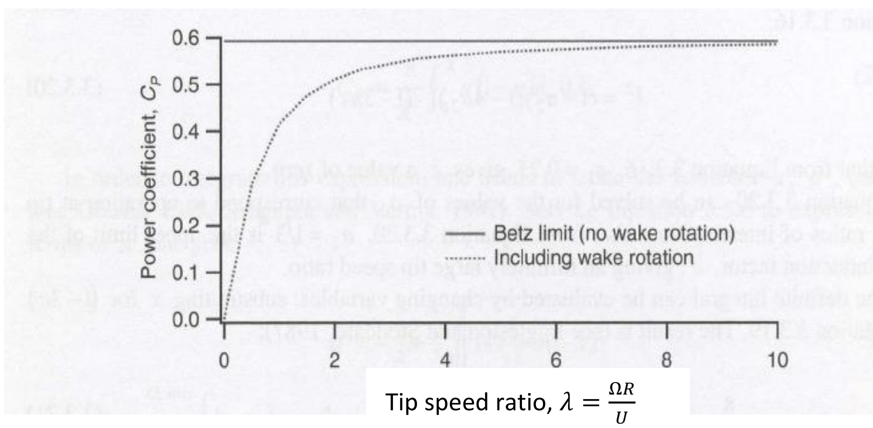

Fig. 5: Theoretical maximum power coefficient as a function of tip speed ratio for an ideal horizontal axis wind turbine, with and without wake rotation

$$\lambda = \frac{\Omega R}{U}$$

U: entering wind speed
Ω: angular velocity of blade
R: radius of rotor

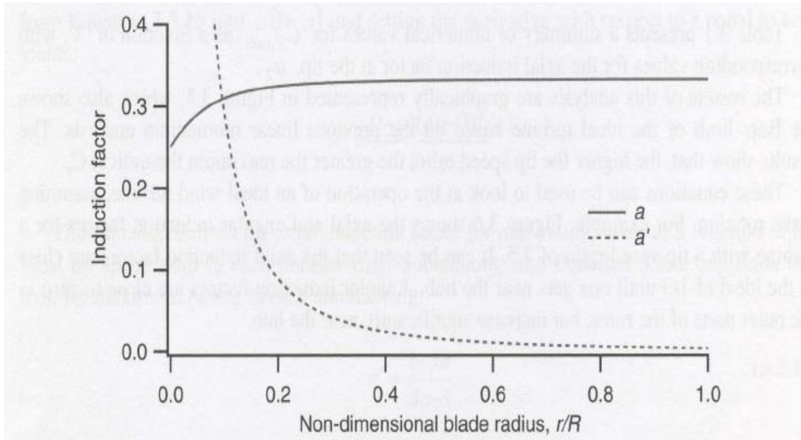

Figure 3.6 Induction factors for an ideal wind turbine with wake rotation; tip speed ratio, λ = 7.5; a, axial induction factor; a', angular induction factor; r, radius; R rotor radius

Fig. 6: Induction factors for an ideal wind turbine with wake rotation: tip speed ratio, $\lambda = 7.5$; a-axial induction factor, a'-angular induction factor, r-radius, and R-rotor radius

Effect of Drag and Blade number on optimum performance

Wilson *et.al.* (1976)[2] obtain the formula for $C_{p,max}$

$$C_{p,max} = \left(\frac{16}{27}\right)\lambda\left\{\lambda + \frac{1.32 + \left(\frac{\lambda - 8}{20}\right)^2}{B^{\frac{2}{3}}}\right\}^{-\mathbf{1}} - \frac{0.57\lambda^2}{\frac{C_l}{C_d}\left(\lambda + \frac{1}{2B}\right)}$$

where, λ is speed ratio $\left(\frac{\Omega R}{U}\right)$, B is number of blades, C_l is lift coefficient of a blade, C_d is the drag coefficient of blade. If no drag is considered, maximum achievable power coefficient is given in Fig. 7.

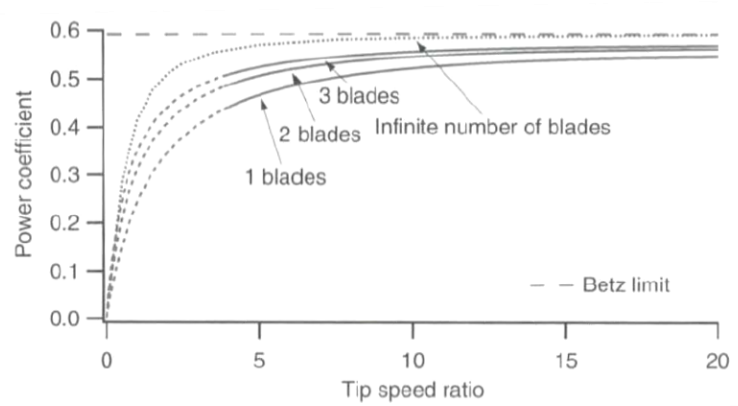

Fig. 7: Maximum achievable power coefficient as a function of number of blades, no drag

[2] Wilson, R. E., Lissaman, P. B. S. and Walker, S. N., 1976, "Aerodynamic performance of wind turbines," Energy research and development administration, ERDA/NSF/04014-7611.

Fig. 8 shows the maximum power achievable as a function of tip speed ratio for 3-blade wind turbine if the lift to drag ratio, $\frac{C_l}{C_d}$ is considered.

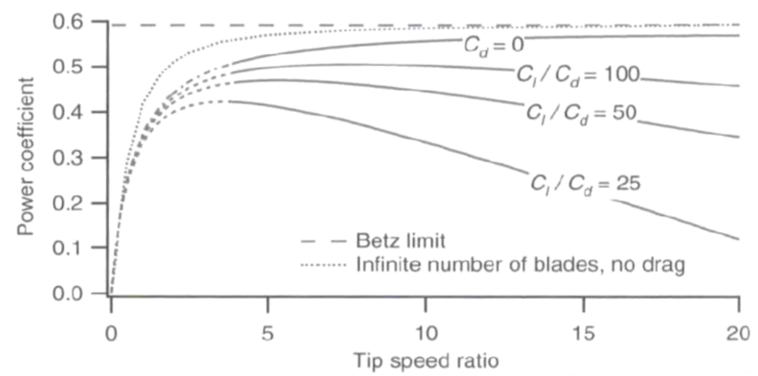

Fig. 8: Maximum achievable power coefficients of a 3-blade optimum rotor as a function of the lift to drag ratio, $\frac{C_l}{C_d}$

2. Fundamental of electromagnetic motor

Magnetic flux density B (Wb/m^2) is related to magnetic field H (A/m) by

$$\underline{B} = \mu \underline{H} \tag{1}$$

where μ is magnetic permeability, and it is conveniently expressed by

$$\mu = \mu_r \mu_0 \tag{2}$$

$\mu_0 = 4\pi \times 10^{-7}\ Wb/(A \cdot m)$, for free space (or in vacuum) μ_r is relative permeability, thus it is non-dimensional.

Non-magnetic materials, $\mu_r = 1$, and ferromagnetic materials, μ_r is very large, $10^3 \sim 10^5$. There are two kinds of ferromagnetic materials, soft and hard (or permanent) magnet, see Fig. 1.

For ferromagnetic materials, Eq. (3) is normally used.

$$\underline{B} = \mu_0 (\underline{H} + \underline{M}) \tag{3a}$$

or

$$= \mu_0 \underline{H} + \underline{M} \tag{3b}$$

(3a) is newer expression then the unit of $\underline{M}$is the same as $\underline{H}$ (i.e., A/m), and (3b) is the older expression, still used by materials scientists. Then the unit of $\underline{M}$ is $\frac{Wb}{m^2} = Tesla$. In Eq. (3), $\underline{M}$ is called as magnetization vector (magnetic dipole moment per unit volume), and it is related to $\underline{H}$ as

$$\underline{M} = \chi_m \underline{H} \tag{4}$$

where χ_m is magnetic susceptibility. From (1), (2), (3a) and (4),

$$\underline{B} = \mu_0\left(\underline{H} + \chi_m \underline{H}\right)$$
$$= \mu_0(1 + \chi_m)\underline{H} \tag{5}$$

Thus, $\mu = \mu_0(1 + \chi_m)$

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m \tag{6}$$

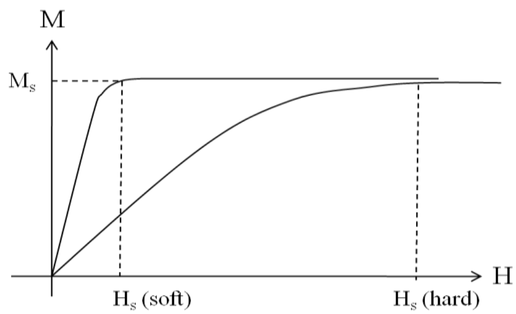

Fig. 1: M-H curve of ferromagnetic materials. Soft magnet has very small H_c, hard magnet has large H_c

Fig. 2: Solenoid with number of turn, N

Let us consider simple solenoid which has N turns. If electric current (I) flows through the solenoid, magnetic flux density $\underline{B}$ is induced and its magnitude B is given by

$$B = \mu I \frac{N}{L} \tag{7}$$

where L is inductance [Henry=H].

Magnetic flux ($\underline{\Phi}$) is defined by

$$\underline{\Phi} = \int \underline{B} \cdot d\underline{A} \tag{8}$$

where $d\underline{A}$ is area element vector. The magnitude of $\underline{\Phi}$ is

$$\Phi = BA$$
$$= \frac{\mu INA}{L} \tag{9}$$

<u>Faraday's law</u>

Electromagnetic force (or voltage) E is given by

$$E = -\frac{d\Phi}{dt} \tag{10}$$

A current flowing in conductor in the presence of magnetic field results in an induced force, acting on the conductor. This is the fundamental property of motors. A conductor which is forced to move through a magnetic field will have a current induced in the conductor. This is the fundamental property of generators. The force in a conductor of incremental length dl, the current (I), magnetic flux density $d\underline{B}$ are related by

$$d\underline{F} = \underline{I}dl \times d\underline{B} \tag{11}$$

It is noted in (11) '×' is vector (or cross) product between two vectors. If vectors $\underline{I}dl$ and $d\underline{B}$ are perpendicular, the force vector which is perpendicular to both $\underline{I}dl$ and $d\underline{B}$, becomes the maximum.

Ampere's law

Current flowing in a conductor induces a magnetic field $\underline{H}$ in the vicinity of the conductor, which is called Ampere's law.

$$\oint \underline{H} \cdot d\underline{l} = I \tag{12}$$

If conductor is solenoid with N turns,

$$V_m = \oint H_s ds = NI \tag{13}$$

H_s is the magnetic field along the magnetic circuit, and V_m is the electromagnetic force. (13) is reduced to

$$V_m = \oint H_s ds = \oint \frac{B}{\mu} ds = \oint \frac{\Phi}{\mu S} ds = \Phi \oint \frac{ds}{\mu S} = \Phi R_m \tag{14}$$

where R_m is magnetic resistance,

$$R_m = \oint \frac{ds}{\mu S} \tag{15}$$

S is the cross sectional area through which Φ passes.

Let us consider electromagnetic (Fig. 3), from (13) and (14),

$$V_m = NI = R_m \Phi$$

$$\therefore \Phi = \frac{NI}{R_m} \tag{16}$$

For Fig. 3 circuit,

$$R_m = \frac{l_y}{\mu_r \mu_0 S_y} + \frac{2g}{\mu_0 S_g} \tag{17}$$

where l_y and S_y are the length of magnetic circuit in the yoke, cross sectional area of the yoke, g is the gap distance, S_g is the cross sectional area of the gap.

For a rotor rotating with angle θ, Fig. 4,

$$R_m = \frac{l_y}{\mu_r \mu_0 S_y} + \frac{2g}{\mu_0 S_g \left(\frac{r\theta}{L}\right)} \tag{18}$$

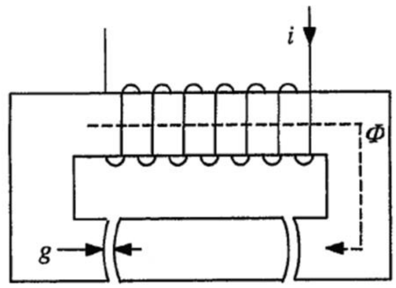

Fig. 3: Simple magnetic device; g-width of air gap, i-current and Φ-magnetic flux

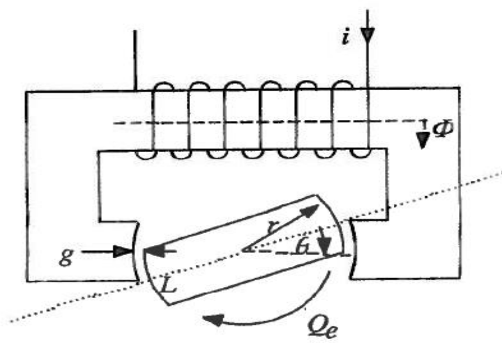

Fig. 4: Simple magnetic torque device; g-width of air gap, i-current, Φ-magnetic flux, L-length of the face of the poles, Q_e-electrical torque, r-radius and θ-rotation angle.