Conclusion: Minimum C_{max} value is

4.6.3 Nawaz, Enscor, and Ham (NEH) Algorithm

Nawaz, Enscor and Ham (NEH) algorithm constructs jobs sequence in iterative manner. Two jobs having largest values of total process times (called total work content) are arranged in a partial sequence one by one. The partial sequence having small value of C_{max} is selected for subsequent iteration. Then, next job from the work content list is picked. This job is alternately placed at all possible locations in the partial sequence. This job is permanently placed at the location where it yields lowest C_{max} value for the partial schedule. In a similar fashion, next job from the work content list is picked, and placed one by one at all possible locations in the partial sequence to find C_{max} value of the partial sequence. This job is permanently placed at the location where partial sequence has minimum C_{max} value. The process is continued till all jobs from the content list are placed in the partial sequence.

NEH algorithm is formally described as under;

Step (1)

Find Total work content (T_j) for each job using expression

$$T_j = \sum_{i=1}^{i=m} p_{ij}$$

Step (2)

Arrange jobs in a work content list according to decreasing values of T_j

Step (3)

Select first two jobs from the list, and form two partial sequences by interchanging the place of the two jobs. Compute C_{max} values of the partial sequences. Out of the two partial sequences, discard the partial sequence having larger value of C_{max}. Call the partial sequence with lower value of C_{max} as *incumbent* sequence

Step (4)

Pick next job from the work content list, and place it at all locations in the *incumbent* sequence. Calculate the value of C_{max} for all the sequences.

Step (5)

Retain the sequence with minimum value of C_{max} as incumbent sequence and, discard all the other sequences.

Step (6)

If there is no job left in the work content list to be added to incumbent sequence, STOP. Otherwise go to step (4).

Example 4.5

Solve $F_3||C_{max}$ problem for the data shown below using NEH algorithm.

	j_1	j_2	j_3	j_4
M_1	6	8	3	4
M_2	5	1	5	4
M_3	4	4	4	2

Solution:

For four jobs, the T_j values are shown in the Table 4.11

Table 4.11 Calculation for T_j values

	j_1	j_2	j_3	j_4
M1	6	8	3	4
M2	5	1	5	4
M3	4	4	4	2
T_j	**15**	**13**	**12**	**10**

The ordered list of jobs according to decreasing T_j values is; $\{j_1, j_2, j_3, j_4\}$

Iteration 1

Since jobs j_1 and j_2 have highest values of T_j, select these two jobs to form partial schedule. The calculations of C_{max} value for partial schedule (j_1 , j_2 , * , *) are shown below in Table 4.12. Note C_{max} = 19 for the partial schedule (j_1 , j_2 , * , *).

Table 1 C_{max} calculations for partial schedule $S_{12\bullet\bullet}$: (j_1, j_2, *, *)

	j_1	j_2	C_1	C_2	C_{max}
M_1	6	8	6	14	
M_2	5	1	11	15	
M_3	4	4	15	19	19

The calculations of C_{max} value for partial schedule (j_2 , j_1 ,* ,*) are shown below in Table 4.13. Note C_{max} = 23 for the partial schedule (j_2 , j_1 ,* ,*).

Table 4.13 C_{max} calculations for partial schedule $S_{21\bullet\bullet}$: (j_2 , j_1 , * , *)

	j_2	j_1	C_1	C_2	C_{max}
M_1	8	6	8	14	
M_2	1	5	9	19	
M_3	4	4	13	23	23

The makespan (C_{max}) values for the partial schedules are;

Table 24.14 Comparison between the two partial sequences

Schedule	C_{max}
$S_{21\bullet\bullet}$: (j_2 , j_1 , *, *)	23
$S_{12\bullet\bullet}$: (j_1 , j_2 , * ,*)	19

Since value of C_{max} is smaller for partial sequence $S_{12\bullet\bullet}$: (j_1 , j_2 , * , *), we further investigate this partial schedule. So partial sequence $S_{21\bullet\bullet}$: (j_2 , j_1 , * , *) is fathomed and not investigated any more.

Iteration 2

Now job j_3 is next in ordered list after jobs j_1 and j_2 with a T_w value of 12. Job j_3 can be placed at three sequence positions in partial sequence (j_1, j_2, *, *).

a) Before job j_1 as follows: New Partial Sequence , $S_{312\bullet}$: (j_3, j_1, j_2, *)
b) After job j_1 as follows: New Partial Sequence , $S_{132\bullet}$: (j_1, j_3, j_2, *)
c) After job j_2 as follows: New Partial Sequence , $S_{123\bullet}$: (j_1, j_2, j_3, *)

Calculations of C_{max} for sequence, $S_{123\bullet}$:(j_1, j_2, j_3, *) are shown below in the following table 4.15.

Table 4.15 C_{max} calculations for partial sequence (j_1 , j_2 , j_3 , *)

	j_1	j_2	j_3	C_1	C_2	C_3	C_{max}
M_1	6	8	3	6	14	17	
M_2	5	1	5	11	15	22	
M_3	4	4	4	15	19	26	26

The Gantt chart of the partial schedule $S_{123\bullet}$:(j_1, j_2, j_3,*) is shown in Figure

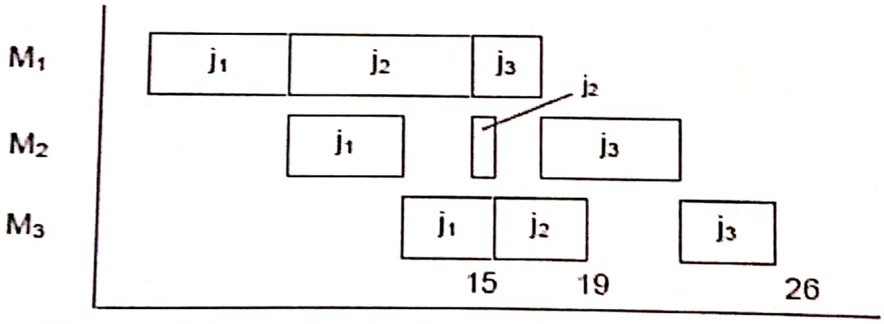

Figure 4.8 Gantt chart for the partial sequence $S_{123\bullet}$: (j_1, j_2, j_3, *)

Note C_{max} = 26 for the partial schedule, $S_{123\bullet}$:((j_1, j_2, j_3,*).

The calculations of C_{max} value for this schedule (j_3,j_1,j_2,*) are shown below in the following table 4.16.

Table 4.16 C_{max} calculations for partial schedule (j_3 , j_1 , j_2 , *)

	j_3	j_1	j_2	C_3	C_1	C_2	C_{max}
M_1	3	6	8	3	9	17	
M_2	5	5	1	8	14	18	
M_3	4	4	4	12	18	22	22

The calculations of C_{max} value for the schedule S_{132*} are shown in the following table 4.17.

Table 34.17 C_{max} calculations for partial schedule S_{132*}: (j_1, j_3, j_2, *)

	j_1	j_3	j_2	C_1	C_3	C_2	C_{max}
M_1	6	3	8	6	9	17	
M_2	5	5	1	11	16	18	
M_3	4	4	4	15	20	24	24

A comparison of the three schedules indicate that schedule S_{312*} : (j_3, j_1, j_2,*) results in minimum C_{max} value, as shown in below table 4.18:

Table 4.18 Comparison of the three partial sequences.

Partial Schedule	C_{max}
S_{123*} (j_1, j_2, j_3, *)	26
S_{312*} (j_3, j_1, j_2, *)	22
S_{132*} (j_1, j_3, j_2,*)	24

Iteration 3

Job j_4 is the last job in the ordered list. Using minimum C_{max} value partial schedule from iteration 2, generate four sequences by inserting job j_4 at four possible locations in partial sequence (j_3, j_1, j_2, *) as follows:

a) Before job j_3 as follows: New Sequence, S_{4312}: (j_4, j_3, j_1, j_2)
b) After job j_3 as follows: New Sequence, S_{3412}: (j_3, j_4, j_1, j_2)
c) After job j_1 as follows: New Sequence, S_{3142}: (j_3, j_1, j_4, j_2)
d) After job j_2 as follows: New Sequence, S_{3124}: (j_3, j_1, j_2, j_4)

The calculations of C_{max} value for this schedule (j_4, j_3, j_1, j_2) are shown below in the following table 4.19

Table 4.19 C_{max} calculations for schedule (j_4, j_3, j_1, j_2)

	j_4	j_3	j_1	j_2	C_4	C_3	C_1	C_2	C_{max}
M_1	4	3	6	8	4	7	13	21	
M_2	4	5	5	1	8	13	18	22	
M_3	2	4	4	4	10	17	22	26	26

The calculations of C_{max} value for this schedule (j_3, j_4, j_1, j_2) are shown below in the following table 4.20

Table 4.20 C_{max} calculations for schedule (j_3, j_4, j_1, j_2)

	j_3	j_4	j_1	j_2	C_3	C_4	C_1	C_2	C_{max}
M_1	3	4	6	8	3	7	13	21	
M_2	5	4	5	1	8	12	18	22	
M_3	4	2	4	4	12	14	22	26	26

The calculations of C_{max} value for this schedule (j_3, j_1, j_4, j_2) are shown below in the following table 4.21

Table 4.21 C_{max} calculations for schedule (j_3, j_1, j_4, j_2)

	j_3	j_1	j_4	j_2	C_3	C_1	C_4	C_2	C_{max}
M_1	3	6	4	8	3	9	13	21	
M_2	5	5	4	1	8	14	18	22	
M_3	4	4	2	4	12	18	20	26	26

The calculations of C_{max} value for this schedule (j_3, j_1, j_2, j_4) are shown below in the following table 4.22

Table 4.22 C_{max} calculations for schedule (j_3, j_1, j_2, j_4)

	j_3	j_1	j_2	j_4	C_3	C_1	C_2	C_4	C_{max}
M_1	3	6	8	4	3	9	17	21	
M_2	5	5	1	4	8	14	18	25	
M_3	4	4	4	2	12	18	19	27	27

The comparison of C_{max} values for the four schedules is presented below in table 4.23

Table 4.23 Comparison of the four partial sequences

Schedule	C_{max}
S_{3124} : (j_3, j_1, j_2, j_4)	27
S_{3124} : (j_4, j_3, j_1, j_2)	26
S_{3412} : (j_3, j_4, j_1, j_2)	26
S_{3142} : (j_3, j_1, j_4, j_2)	26

The NEH method yields three alternate schedules with a minimum makespan of 26. Clearly, NEH provides more elaborate results as compared to CDS or Slope heuristic.