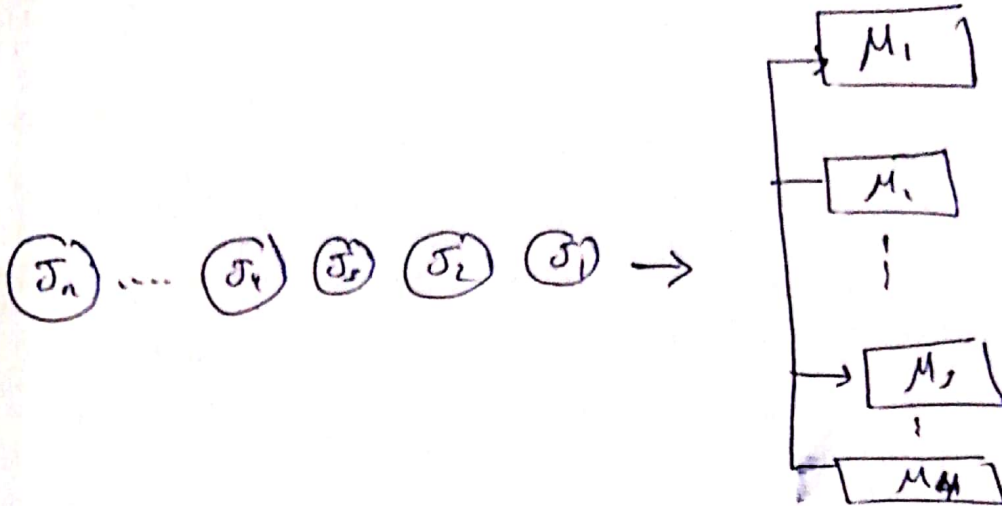


Parallel machines

ماتر التوازي

①

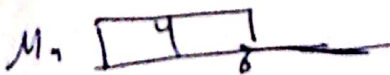
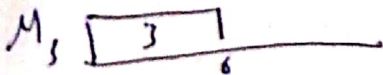
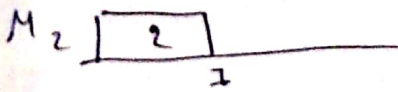
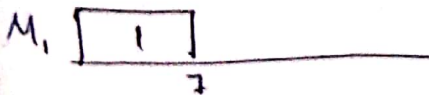


$nP || C_{max}$ مسألة التوازي

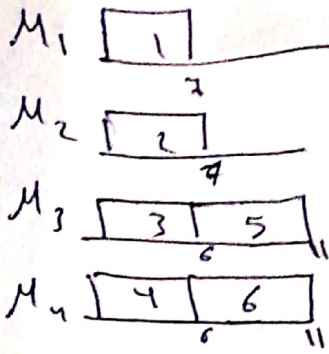
	1	2	3	4	5	6	7	8	9
P_{ij}	7	7	6	6	5	5	4	4	4

$P || C_{max}$

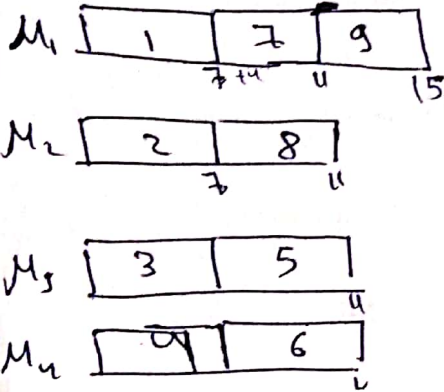
الشيء الذي يجب ان نتذكره في هذه المسألة
 نكتب LPT + P_{ij}



Machine (M)	Job assigned	start time S_j	processing time P_{ij}	Completion time (C_j)
M_1	1	0	7	7
M_2	2	0	7	7
M_3	3	0	6	6
M_4	4	0	6	6



Job J	Machine (M)	start time S_j	processing time P_{ij}	Completion time C_j
J_5	M_3	6	5	11
J_6	M_4	6	5	11



Job (j)	Machine (M)	Start time S_j	processing time P_{ij}	Completion time C_j
J_7	M_1	7	4	11
J_8	M_2	7	4	11

Job (j)	Machine (M)	starting time S_j	processing time P_{ij}	Completion time C_j
J_1	M_1	0	7	7
J_2	M_2	0	7	7
J_3	M_3	0	6	6
J_4	M_4	0	6	6
J_5	M_3	6	5	11
J_6	M_4	6	5	11
J_7	M_1	7	4	11
J_8	M_2	7	4	11
J_9	M_1	11	4	15

$C_{max} = 15$

The following relationship can be used to find how far the solution obtained by the LPT rule is far from an optimal solution

$$\frac{C_{max}(LPT)}{C_{max}(OPT)} \leq \frac{4}{3} - \frac{1}{3m}$$

Note Is the C_{max} value obtained in above example optimal? if not, what is the optimal solution

جواب For finding optimality of schedules, the following ratio between $C_{max}(OPT)$ and $C_{max}(LPT)$ is observed

$$\frac{C_{max}(LPT)}{C_{max}(OPT)} \leq \frac{4}{3} - \frac{1}{3m}$$

$$\because m=4, \quad \frac{C_{max}(LPT)}{C_{max}(OPT)} \leq 1.25$$

$$\because C_{max}(LPT) = 1.25 \times C_{max}(OPT)$$

$$\Rightarrow C_{max}(OPT) = \frac{C_{max}(LPT)}{1.25} = \frac{15}{1.25} = \underline{\underline{12}}$$

المسألة هي أن نحصل على قيمة C_{max} (optimal) من خلال تطبيق قاعدة Load Balancing heuristic as follows,

⊗ Let T_w = Total work content of all jobs in the problem.

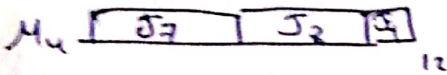
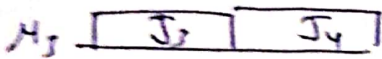
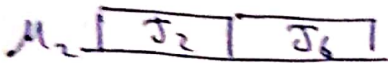
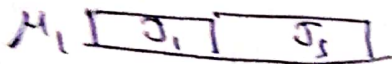
⊗ tentative load per machine may be estimated by taking the ratio of T_w and m as follows:-

$$\text{Load per machine} = \frac{T_w}{m}$$

$$T_w = \sum_{j=1}^9 p_j = 48, \text{ and } m=4$$

optimal schedule using load Balancing heuristic

Machine	Jobs	Total Time
M ₁	J ₁ , J ₅	7+5=12
M ₂	J ₂ , J ₆	7+5=12
M ₃	J ₃ , J ₄	6+6=12
M ₄	J ₇ , J ₈ , J ₉	4+4+4=12



Gantt chart for optimal solution