Figure 3.5 Critical Path (j_3-j_4-j_5-j_8-j_7) for the precedence diagram.

MINIMIZATION OF MAKESPAN WITH P_j =1 AND PRECEDENCE PROBLEM
(P_m / P_j = 1, tree / C_{max})

This parallel machine-scheduling problem pertains to jobs having precedence relationship represented by a *tree*. All the jobs on the tree have process time equal to unity. If the jobs have precedence relationship described either by *intree* or by *outtree* (shown below), then CP rule can be applied to minimize C_{max}.

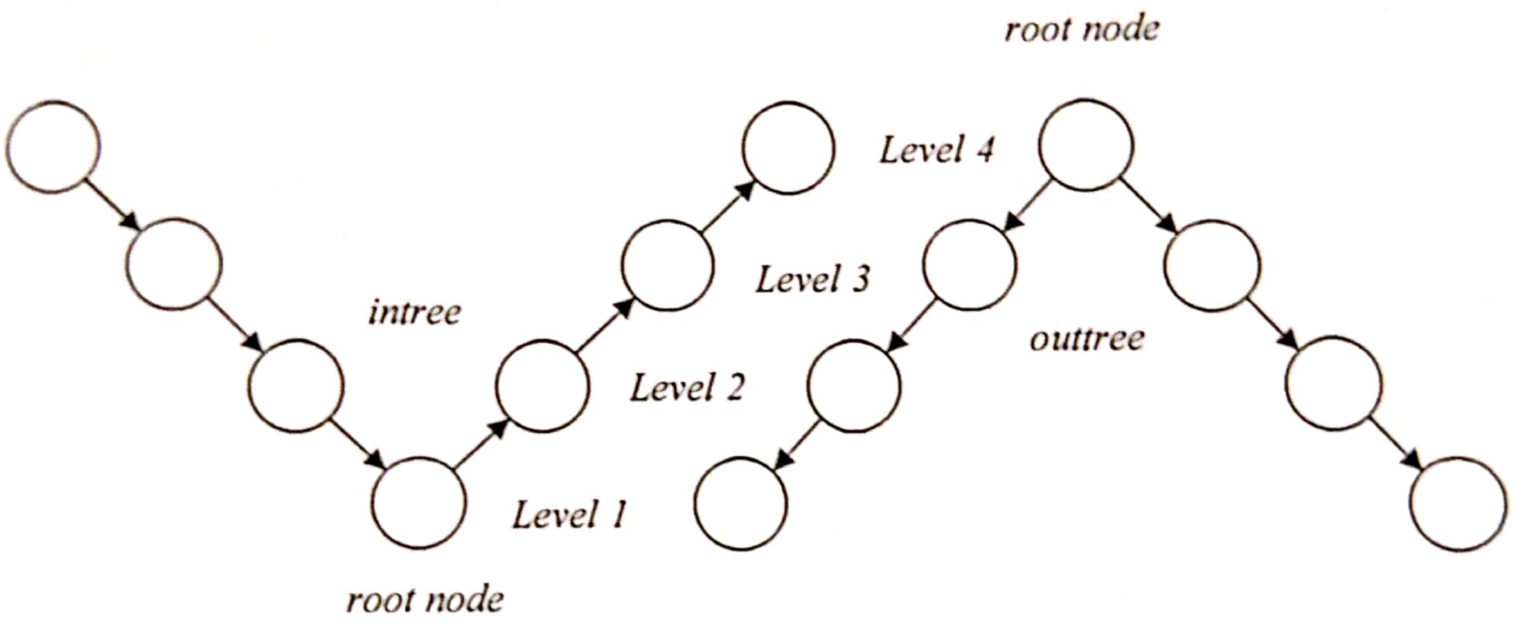


Figure 3.6 Relationship of *intree* and *outtree*

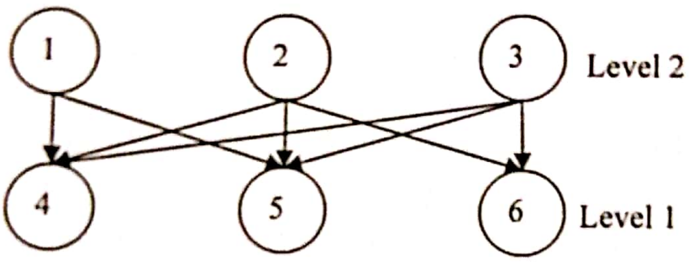
3.4.1 CP Rule

The root node for *intree* precedence graph is at lowest level (Level 1), whereas root node for *outtree* precedence graph is at highest level (Level 5). CP rule imparts high priority to a job having longest string of jobs in the precedence graph. This means that a job having highest level in the precedence graph will be schedules first. Hence CP rule is in fact ***Highest Level First*** rule. This means that root node of *outtree* precedence graph will have highest priority and, root node of *intree* precedence graph will be given least priority. Note that upper bound (UB) on the worst case performance of CP rule is given by the following formula.

$$\frac{C_{max}(CP)}{C_{max}(OPT)} \leq \frac{4}{3}$$

Example 3.4

For the $Pm \mid p_j = 1, tree \mid C_{max}$, precedence graph shown below, apply CP rule and find C_{max}. The production shop has two machines in parallel.



Solution:

Using CP rule, first select jobs with Highest Level, hence the jobs in candidate set at time t=0 are from Level 2 jobs; i.e., jobs $\{j_1, j_2, j_3\}$. Schedule job 1 on machine 1 and job 2 on machine 2. At time 1, only job 3 is schedulable. Hence, schedule job 3 on machine 1 at time 1. At time 2, all the jobs at level 1 are schedulable. Jobs at Level 1 include $\{j_4, j_5, j_6\}$. However, only two jobs can be scheduled. So, schedule job 4 and 5 on machine 1 and 2 respectively at time 2. Finally, at time 3 schedule job 6 on machine 2. The complete schedule is shown in Gantt chart (Figure 3.7).

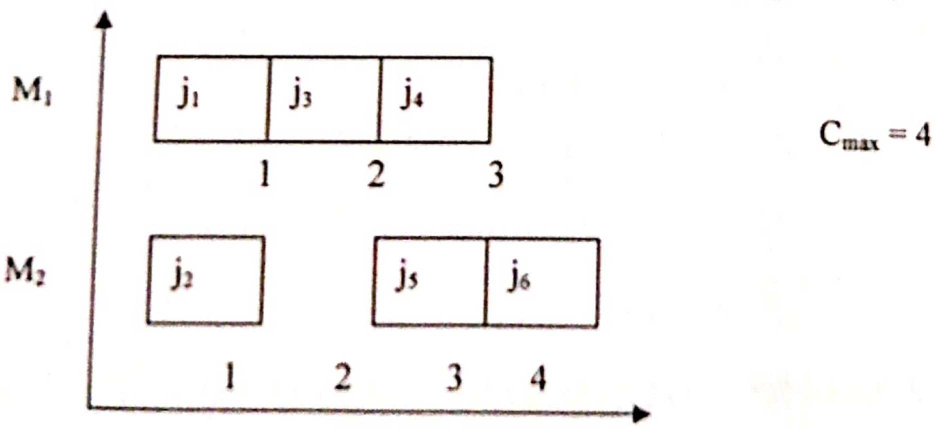


Figure 3.7 Gantt chart for schedule generated by CP Rule.

3.4.2 LNS Rule

The LNS rule is stated as follows:

"Find the number of successors (NS) of all jobs in the precedence graph. While scheduling jobs on parallel machines, give priority to jobs having largest number of successors."

Example 3.5

Solve the problem in Example 3.4 using LNS rule.

Solution

The number of successors for each job in example 3.5 according to precedence graph is shown in the following table:

Job (j)	1	2	3	4	5	6
p_j	1	1	1	1	1	1
NS	2	3	3	0	0	0

At time t=0,

Assign jobs j_2 and j_3 to machines M_1 and M_2.

At time t=1,

Schedule job j_1 on machine M_1, machine M_2 is also free and ready at time t=1. All the three jobs $\{j_3, j_4, j_5\}$ have NS=0. But only job j_6 is schedulable at t=1 because it satisfies the precedence constraint. Hence, schedule job j_6 at time t=1.

At time t=2,

The remaining unscheduled jobs $\{j_4, j_5\}$ are schedulable. So, schedule jobs j_4 and j_5 on machines M_1 and M_2 respectively at time t=2. Note, the makespan (C_{max}) of this schedule is 3 time units.

The complete schedule is presented in Gantt chart (Figure 3.8)

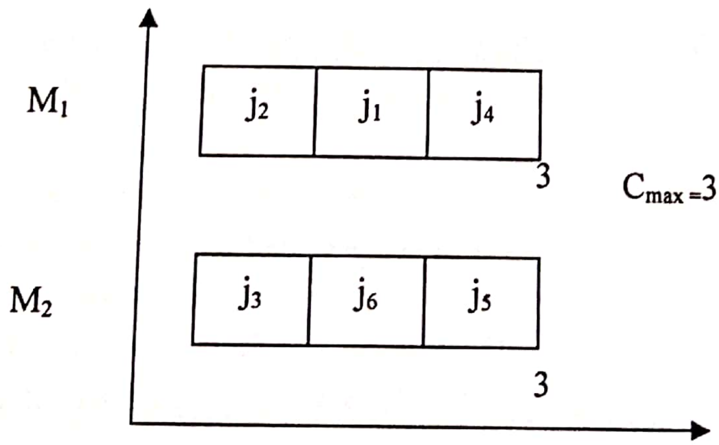


Figure 3.8 Gantt chart for Optimal Schedule generated by LNS rule.