3.3 MINIMIZATION OF MAKESPAN WITH PRECEDENCE (P_m / $prec$ / C_{max})

This scheduling problem occurs when jobs have precedence relationship among them. A precedence relationship diagram shows the precedence relationship. The scheduling calculations are carried out in two steps. In first step, forward pass calculations are made to find out earliest completion times of jobs as under:

Let,

$$C_j' = \text{Earliest Completion Time of job } j$$

$$C_j' = \max_{i \in \Psi} (C_i') + p_j$$

Where, C_i' is the earliest completion time of job i that belongs to set ψ. Set ψ contains predecessor jobs for job j.

To find critical path on the precedence network, latest completion times of all jobs are calculated as follows:

Let,

$$C_j'' = \text{Latest Completion Time of job } j$$

$$C_j'' = \min_{k \in \Omega} (C_k'' - p_k)$$

Where,

$$C_k'' = \text{Latest completion time of job } k \text{ that belongs to set } \Omega$$

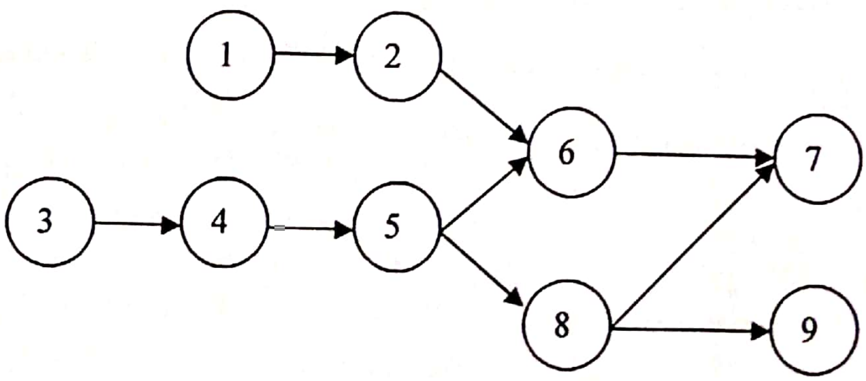
Set Ω contains successor jobs for job j. All the jobs without any successor are assigned a value equal to C_{max} for backward pass calculations.

Example 3.3

A parallel shop has nine jobs with process times as follows:

Job (j)	1	2	3	4	5	6	7	8	9
p_j	4	9	3	3	6	8	8	12	6

The job's precedence constraints are shown in the following precedence diagram.



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Assume infinite number of machines. Use directed graph technique and find;
1) Makespan 2) Critical Path

Solution:

The earliest completion times C'_j of all jobs are shown below in Table 6.

Table 3.6 Earliest completion times of all jobs.

Job (j)	1	2	3	4	5	6	7	8	9
C'_j	4	13	3	6	12	21	32	24	30

The earliest completion times are shown below in Figure 4 on the nodes of the precedence network.

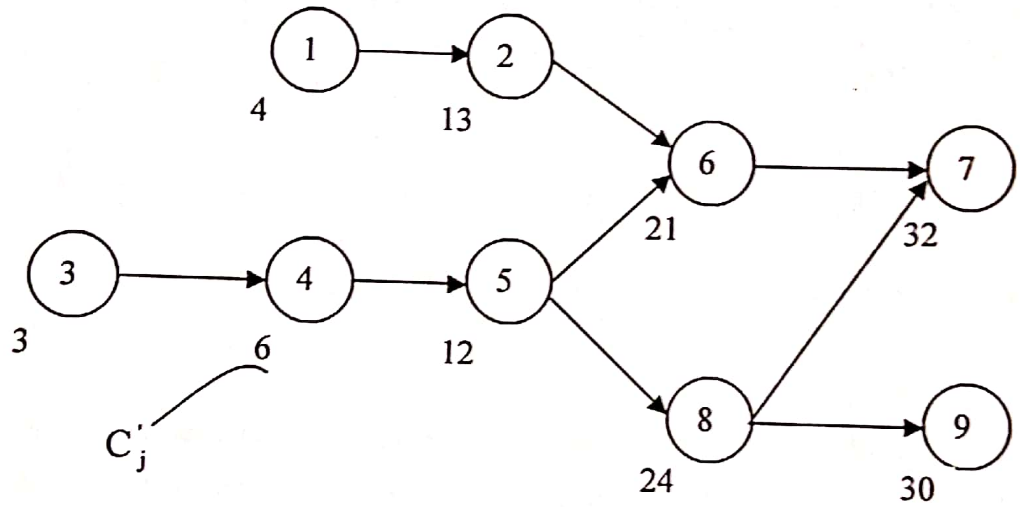

Figure 3.4 Precedence diagram showing earliest completion times.

Job 7 has maximum completion time, which is equal to 32. Hence, the value of C_{max} is equal to 32. To find critical path on the precedence network, latest completion times of all jobs is calculated as follows:

All the jobs without any successor are assigned a value equal to C_{max} for backward pass calculations. In the example problem, jobs 7 and 9 have no successor. Hence, jobs 7 and 9 are assigned latest completion times equal to 32. The latest completion time of job 8 is calculated as follows:

$$C''_8 = \min\{C''_7 - p_7, C''_9 - p_9) = \min\{32-8, 32-6\} = \min\{24, 26\} = 24$$

The latest completion times C''_j of all jobs are shown below in Table 3.7.

Table 3.7 Latest completion times of all jobs.

Job (j)	1	2	3	4	5	6	7	8	9
C''_j	5	16	3	6	12	24	32	24	32

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The earliest and latest completion times of all jobs are shown below in Figure 3.5 on the corresponding nodes of the precedence network. The nodes having equal values of earliest and latest completion times define the critical jobs as well as critical path. For example, jobs j_3, j_4, j_5, j_8 and j_7 have equal values of earliest and latest completion times. The arcs joining these nodes form critical path as shown in Figure 5.

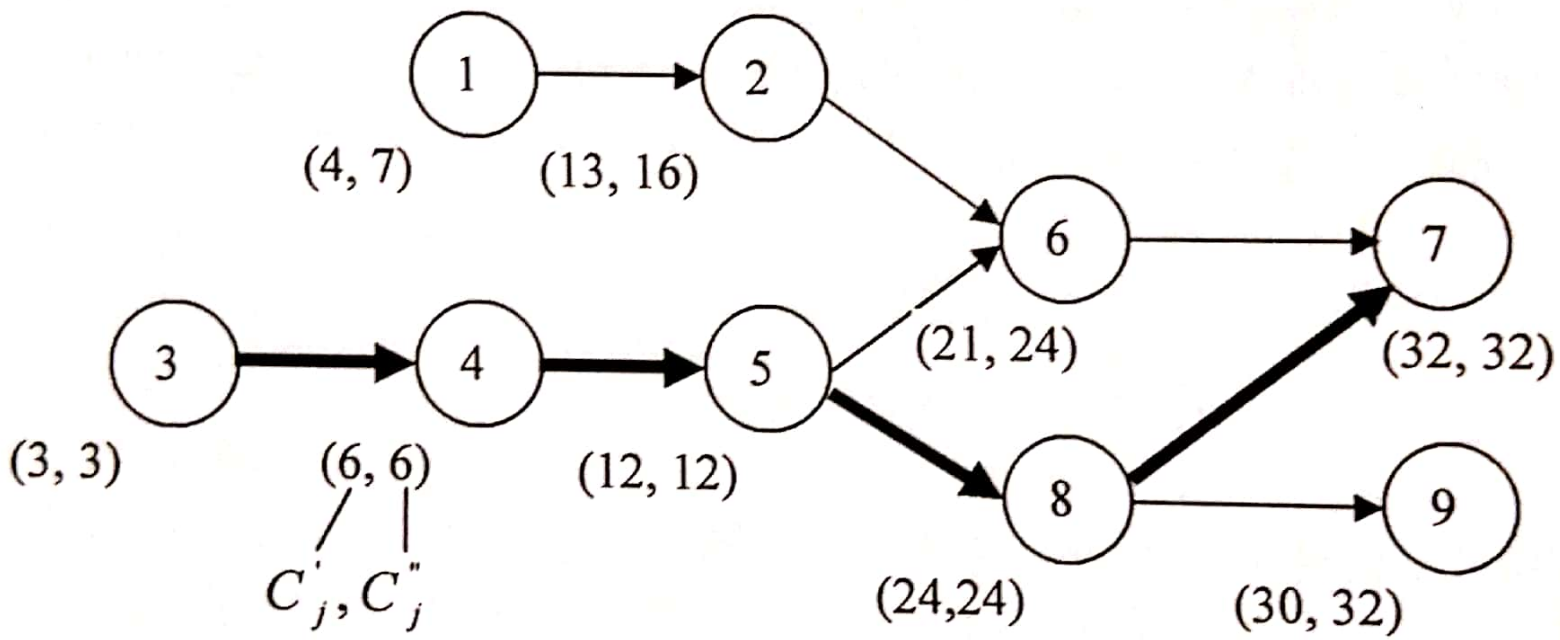

Figure 3.5 Critical Path (j_3-j_4-j_5-j_8-j_7) for the precedence diagram.

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