Lenz's Law

The direction of the induced current is determined by Lenz's law:

The induced current produces magnetic fields which tend to oppose the change in magnetic flux that induces such currents.

To illustrate how Lenz's law works, let's consider a conducting loop placed in a magnetic field. We follow the procedure below:

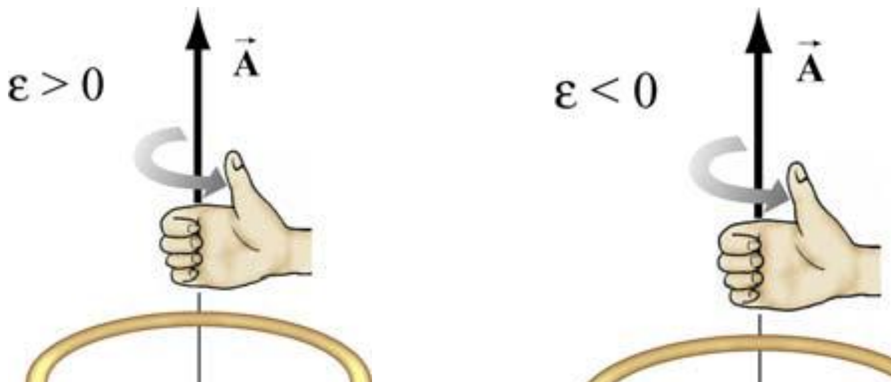
Define a positive direction for the area vector A.

Assuming that B is uniform, take the dot product of B and A. This allows for the determination of the sign of the magnetic flux Φ_B

Obtain the rate of flux change $d\Phi_B/dt$ by differentiation. There are three possibilities:

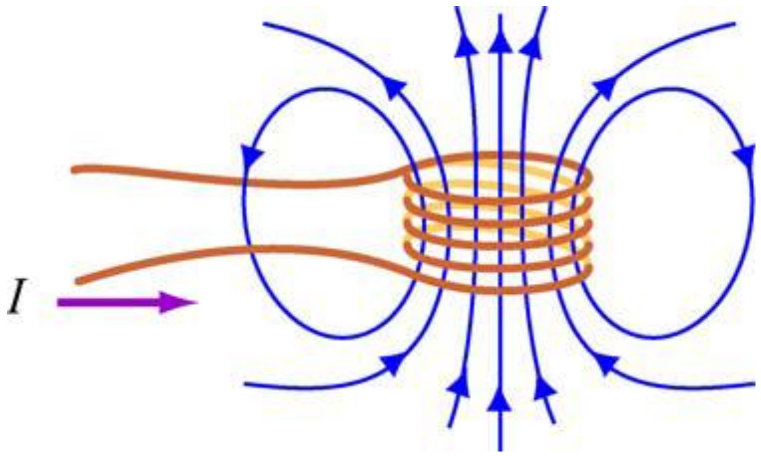
$$\frac{d\Phi_B}{dt}: \begin{cases} >0 & \Rightarrow \text{ induced emf } \varepsilon < 0 \\ <0 & \Rightarrow \text{ induced emf } \varepsilon > 0 \\ =0 & \Rightarrow \text{ induced emf } \varepsilon = 0 \end{cases}$$

Determine the direction of the induced current using the right-hand rule. With your thumb pointing in the direction of A, curl the fingers around the closed loop. The induced current flows in the same direction as the way your fingers curl if ε>0, and the opposite direction if ε<0, as shown in Figure



Self-Inductance

Consider a coil consisting of *N* turns and carrying current *I* in the counterclockwise direction, as shown in Figure. If the current is steady, then the magnetic flux through the loop will remain constant. However, suppose the current *I* changes with time,
then according to Faraday’s law, an induced emf will arise to oppose the change. The induced current will flow clockwise if $dI/dt>0$ and counterclockwise if $dI/dt<0$. The property of the loop in which its own magnetic field opposes any change in current is called “self-inductance,” and the emf generated is called the self-induced emf or back emf, which we denote as ε_L. All current-carrying loops exhibit this property. In particular, an inductor is a circuit element (symbol ) which has a large self-inductance.



Mathematically, the self-induced emf can be written as

$$\varepsilon_L = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}\iint \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}}$$

and is related to the self-inductance *L* by

$$\varepsilon_L = -L\frac{dI}{dt}$$

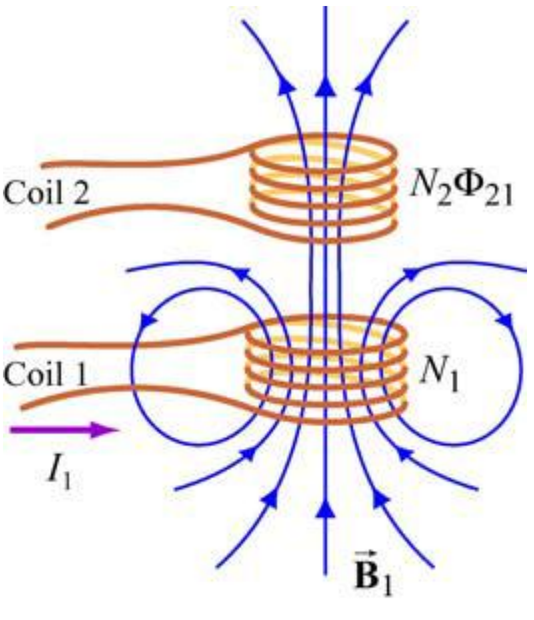
The two expressions can be combined to yield

$$L = \frac{N\Phi_B}{I}$$

Physically, the inductance *L* is a measure of an inductor's "resistance" to the change of current; the larger the value of *L*, the lower the rate of change of current.

Mutual Inductance

Suppose two coils are placed near each other, as shown in Figure



The first coil has N_1 turns and carries a current I_1 which gives rise to a magnetic field $\mathbf{B}_1$. Since the two coils are close to each other, some of the magnetic field lines through coil 1 will also pass through coil 2. Let Φ_{21} denote the magnetic flux through one turn of coil 2 due to I_1. Now, by varying I_1 with time, there will be an induced emf associated with the changing magnetic flux in the second coil:

$$\varepsilon_{21} = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d}{dt} \iint_{\text{coil 2}} \vec{\mathbf{B}}_1 \cdot d\vec{\mathbf{A}}_2$$

The time rate of change of magnetic flux Φ_{21} in coil 2 is proportional to the time rate of change of the current in coil 1:

$$N_2 \frac{d\Phi_{21}}{dt} = M_{21} \frac{dI_1}{dt}$$

where the proportionality constant M_{21} is called the mutual inductance. It can also be written as

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}$$

The SI unit for inductance is the henry (H):

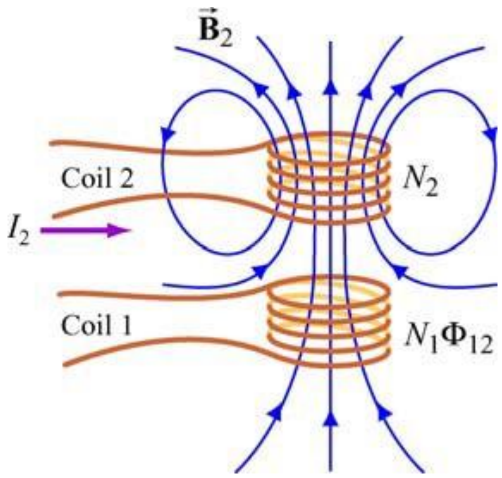
$$1 \text{ henry} = 1 \text{ H} = 1 \text{ T} \cdot \text{m}^2/\text{A}$$

We shall see that the mutual inductance M_{21} depends only on the geometrical properties of the two coils such as the number of turns and the radii of the two coils.

In a similar manner, suppose instead there is a current I_2 in the second coil and it is varying with time . Then the induced emf in coil 1 becomes

$$\varepsilon_{12} = -N_1 \frac{d\Phi_{12}}{dt} = -\frac{d}{dt} \iint_{\text{coil 1}} \vec{\mathbf{B}}_2 \cdot d\vec{\mathbf{A}}_1$$

and a current is induced in coil 1.



Changing current in coil 2 produces changing magnetic flux in coil 1.

This changing flux in coil 1 is proportional to the changing current in coil 2,

$$N_1 \frac{d\Phi_{12}}{dt} = M_{12} \frac{dI_2}{dt}$$

where the proportionality constant M_{12} is another mutual inductance and can be written as

$$M_{12} = \frac{N_1\Phi_{12}}{I_2}$$