نعوض في (1) و (2)

$-8 + 3B + C = 0$

$-2 + B + C = 1$

$\Rightarrow B + C = 3$

$-3B \mp C = \mp 8$ بالطرح

$-2B = -5 \Rightarrow B = \frac{5}{2}$

$C = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$

نعوض في (*)

$$L^{-1}\left\{\frac{P^2-6}{P(P+1)(P+3)}\right\} = L^{-1}\left\{\frac{-2}{P}\right\} + L^{-1}\left\{\frac{5/2}{P+1}\right\} + L^{-1}\left\{\frac{1/2}{P+3}\right\}$$

$$= -2L^{-1}\left\{\frac{1}{P}\right\} + \frac{5}{2}L^{-1}\left\{\frac{1}{P+1}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{P+3}\right\}$$

$$= -2 + \frac{5}{2}e^{-t} + \frac{1}{2}e^{-3t}.$$

Exercises

a) $\frac{1}{S^2(S^2+1)}$ b) $\frac{16}{S(S^2+4)^2}$

c) $\frac{1}{S^2(S^2+1)}$ d) $\frac{S-4}{(S-4)^2+9}$

Solve differential equation by Laplace & inverse transformation

أن اهم تطبيقات لابلاس ومعكوسة هو حل معادلات تفاضلية خطية ذات معاملات ثابتة حيث يتطلب تحويل لابلاس تحويل المعادلة التفاضلية الخطية الى معادلة جبرية بدلالة $\bar{y}(P)$ ويتم ذلك باستخدام المشتقات

$$\frac{d^n y}{dx^n}, \frac{d^{n-1} y}{dx^{n-1}}, \ldots, \frac{dy}{dx}$$

Theorem: If $y^{(n)}(x), y^{(n-1)}(x), \ldots, y'(x)$ functions then the Laplace transformation is

$$L\{y^{n}(x)\} = p^n \bar{y}(p) - p^{n-1} y(0) - p^{n-2} y'(0) - \cdots - p\, y^{(n-2)} - y^{(n-1)}(0)$$

Example: $\frac{dy}{dx} + 2y = \cos x$, $y(0) = 0$ Find Solution of equation by Laplace transformation.

$$\mathcal{L}\{y' + 2y\} = \mathcal{L}\{\cos x\}$$

$$\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{\cos x\}$$

$$p\,\bar{y}(p) - y(0) + 2\,\bar{y}(p) = \frac{p}{p^2+1}$$

$$p\,\bar{y}(p) - 0 + 2\,\bar{y}(p) = \frac{p}{p^2+1}$$

$$\bar{y}(p)(p+2) = \frac{p}{p^2+1}$$

$$\bar{y}(p) = \frac{p}{p^2+1} * \frac{1}{p+2} = \frac{p}{(p^2+1)(p+2)}$$

$$\mathcal{L}^{-1}\{\bar{y}(p)\} = \mathcal{L}^{-1}\left\{\frac{p}{(p^2+1)(p+2)}\right\}$$ ناخذ $\mathcal{L}^{-1}$

تجزئة الكسور

$$\frac{p}{(p^2+1)(p+2)} = \frac{Ap+B}{p^2+1} + \frac{C}{p+2}$$

$$\frac{(AP+B)(P+2)+C(P^2+1)}{(P^2+1)(P+2)} = \frac{AP^2+BP+2AP+2B+CP^2+C}{(P^2+1)(P+2)}$$

معامل $P^2 \rightarrow A + C = 0 \quad — ① \Rightarrow A = -C$

$P \rightarrow 2A + B = 1 \quad ②$

$2B + C = 0 \quad ③ \Rightarrow B = -\frac{1}{2}C$

$2\cdot(-C) + (-\frac{1}{2}C) = 1$ نعوض في ②

$-2C - \frac{1}{2}C = 1 \Rightarrow \frac{-5}{2}C = 1 \Rightarrow C = \frac{-2}{5}$

$A = \frac{2}{5} \quad , B = \frac{1}{5}$

$$\therefore \mathcal{L}^{-1}\left\{\frac{P}{(P^2+1)(P+2)}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{2}{5}P+\frac{1}{5}}{P^2+1} + \frac{-\frac{2}{5}}{P+2}\right\}$$

$$y(x) = \mathcal{L}^{-1}\{\bar{y}(P)\} = \cancel{\frac{2}{5}\mathcal{L}^{-1}\left\{\frac{P}{P^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{2/5}{P+2}\right\}}$$

$$= \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{P}{P^2+1}\right\} + \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{P^2+1}\right\} - \frac{2}{5}\left\{\frac{1}{P+2}\right\}$$

$$= \frac{2}{5}\cos x + \frac{1}{5}\sin x - \frac{2}{5}e^{-2x}$$

Q) Solve the differential equation by Laplace transformation and Inverse of Laplace -

1) $y'' - 4y' + 3y = 6e^{4x}$, $y(0) = 2$, $y'(0) = 6$

2) $y'' - 2y' + 2y = 12x\,e^{x}$ $y(0) = 0$, $y'(0) = 1$

3) $y'' + 4y = 10\sin 3x - 3\cos 3x$, $y(0) = 2$, $y'(0) = -4$

4) $y'' + 9y = 6\cos 3x$ $y(0) = 0$, $y'(0) = 3$

So/ $\mathcal{L}\{y'' - 4y' + 3y\} = \mathcal{L}\{6e^{4x}\}$

$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 3\mathcal{L}\{y\} = \mathcal{L}\{6e^{4x}\}$

$P^2\bar{y}(P) - Py(0) - y'(0) - 4(P\bar{y}(P) - y(0)) + 3\bar{y}(P) = \frac{6}{P-4}$

$P^2\bar{y}(P) - P\cdot 2 - 6 - 4(P\bar{y}(P) - 2) + 3\bar{y}(P) = \frac{6}{P-4}$

$\bar{y}(P^2 - 4P + 3) - 2P + 2 = \frac{6}{P-4}$

$\bar{y}(P)(P^2 - 4P + 3) = \frac{6}{P-4} + (2P - 2)$

$= \frac{6 + (P-4)(2P-2)}{(P-4)} = \frac{2(P^2 - 5P + 7)}{(P-4)}$

$\bar{y}(P) = \frac{2(P^2 - 5P + 7)}{(P-4)} * \frac{1}{(P^2 - 4P + 3)}$

$= \frac{2(P^2 - 5P + 7)}{(P-4)(P-3)(P-1)}$

$$\therefore \mathcal{L}^{-1}\{\bar{y}(P)\} = \mathcal{L}^{-1}\left\{\frac{2(P^2-5P+7)}{(P-4)(P-3)(P-1)}\right\}$$

تجزئة الكسور

$$\frac{A}{P-4} + \frac{B}{P-3} + \frac{C}{P-1}$$

$$\Rightarrow \frac{A(P-3)(P-1) + B(P-4)(P-1) + C(P-4)(P-3)}{(P-4)(P-3)(P-1)}$$

$$= \frac{A(P^2-4P+3) + B(P^2-5P+4) + C(P^2-7P+12)}{(P-4)(P-3)(P-1)}$$

مساواة

$$P^2 \Rightarrow A + B + C = 2$$

$$P \rightarrow -4A - 5B - 7C = -10$$

$$P^0 \rightarrow 3A + 4B + 12C = 14$$

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