Equations of Straight Line معادلة المماس

ملاحظة: لإيجاد معادلة المماس لمنحني الدالة نطبق ما يلي

$$y - y_0 = m(x - x_0)$$ حفظ

* لأيجاد معادلة المماس يجب توفر ميل ونقطة.

Ex① Find the equation of the Tangent Line to the curve $y = \cos\sqrt{x}$ at $(\pi^2, 3)$

$$\frac{dy}{dx} = -\sin\sqrt{x} * \frac{1}{2\sqrt{x}} = -\frac{1}{2}\,\frac{\sin\sqrt{x}}{\sqrt{x}}$$

$$m = \frac{dy}{dx}(x_0, y_0)$$ حفظ

$$\therefore \frac{dy}{dx} = m = -\frac{1}{2}\,\frac{\sin\sqrt{x}}{\sqrt{x}}$$

$$m = \frac{dy}{dx}(\pi^2) = -\frac{1}{2}\,\frac{\sin\pi}{\pi} = -\frac{1}{2}\,\frac{0}{\pi} = 0 \qquad (\sin\pi) = 0$$

$$\therefore m = 0$$

$$\therefore y - y_0 = m(x - x_0) \Rightarrow y - 3 = 0(x - \pi^2)$$

$$y - 3 = 0 \Rightarrow$$

$\therefore y - 3 = 0$ The equation of tangent line

ملاحظة: عندما يطلب معادلة المماس لمنحني الدالة وقال أنها تمر بالنقطتين $P_1(x_1, y_1)$ ، $P_2(x_2, y_2)$ فإننا نستخرج الميل كالتالي.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ حفظ

Ex) find the equation for the Line that passes through points $P_1(-2,0)$ and $P_2(2,-2)$

Sol/ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-0}{2-(-2)} = \frac{-2}{4} = -\frac{1}{2}$

$\therefore y - y_0 = m(x - x_0)$

أول نقطة تحقق معادلة المطلوب هي $(-2,0)$

$\therefore y - 0 = -\frac{1}{2}(x-(-2)) \Rightarrow y = -\frac{1}{2}x - 1$

نضرب طرفي المعادلة بـ 2

$2y = -x - 2$

$\Rightarrow 2y + x + 2 = 0$ equation of tangent Line

Ex) find the equation of the tangent to curve $y = (1+\sqrt[3]{x})^3$ at $x=1$

Sol/ to find $y_0 \Rightarrow y(1) = (1+\sqrt[3]{1})^3 = (2)^3 = 8$

$\therefore x_0 = 1, \quad y_0 = 8 \Rightarrow (1,8)$

$\frac{dy}{dx} = \cancel{3}(1+\sqrt[3]{x})^2 \cdot \frac{1}{\cancel{3}} x^{-\frac{2}{3}} = \frac{(1+\sqrt[3]{x})^2}{x^{\frac{2}{3}}}$

$\therefore m = y(1,8) = \frac{(1+\sqrt[3]{1})^2}{(1)^{\frac{2}{3}}} = \frac{(2)^2}{1} = 4$

$\therefore y - y_0 = m(x - x_0) \Rightarrow y - 8 = 4(x-1)$

$y - 8 = 4x - 4 \Rightarrow y - 4x - 4 = 0$ equation of the tangent

ملاحظة: ① المستقيمات المتوازيات لها نفس الميل معكوس

② اما المستقيمات المتعامدات فان حاصل ضرب ميلها = -1

اي إن ميل الثاني = معكوس السالب لميل المستقيم الأول

وبالعكس. i.e $m_2 = \frac{-1}{m_1}$

ex) Find the equation for the line through

① $P(2,1)$ Parallel to $L: y = x^2 + 2$

② an equation for the line through P perpendicular to $L: y = x^2 + 2$

Sol/ $\frac{dy}{dx} = 2x \Rightarrow m_1 = 2x \Rightarrow m_1(2,1) = 4$ $^{2\times 2}$

$y - y_0 = m_1(x - x_0) \Rightarrow y - 1 = 4(x - 2)$

$y - 1 = 4x - 8$

$y - 4x + 7 = 0$

② Since $y = x^2 + 2$ is Perpendicular to L then

$m_2 = \frac{-1}{m_1} = \frac{-1}{4}$

$\therefore (y - y_0) = m_2(x - x_0) \Rightarrow y - 1 = -\frac{1}{4}(x - 2)$

$(y - 1 = -\frac{1}{4}x + \frac{1}{2}) * 4$

$4y - 4 = -x + 2 \Rightarrow 4y + x - 6 = 0$

ملاحظة:- عندما يعطي أو يطلب الزاوية (θ) في هذا الموضوع فأن ($m = \tan\theta$)

Ex) Find the equation of the tangent through the point $p(1,4)$ with the angle of in $\theta = 60$

Sol/ $m = \tan\theta \Rightarrow m = \tan 60 = \sqrt{3}$

$\therefore y - y_0 = m(x - x_0) \Rightarrow y - 4 = \sqrt{3}(x - 1)$

$\Rightarrow y - \sqrt{3}x - 4 + \sqrt{3} = 0$ equation of the tangent Line

ex) Find the equation of the straight line through $A(7,5)$ perpendicular to the line AB whose the equation is $3x + 4y - 16 = 0$

Sol/ $3x + 4y - 16 = 0 \Rightarrow 4y = 16 - 3x$

$\Rightarrow y = 4 - \frac{3}{4}x \Rightarrow \frac{dy}{dx} = -\frac{3}{4}$

$\therefore$ Since perpendicular then $m = \frac{4}{3}$

الميل العمود على المستقيم

$\therefore y - y_0 = m(x - x_0) \Rightarrow (y - 5) = \frac{4}{3}(x - 7)$

$\Rightarrow (y - 5 = \frac{4}{3}x - \frac{28}{3}) * 3$

$\Rightarrow 3y - 15 = 4x - 28 \Rightarrow 3y - 4x + 13 = 0$

ex) Find the equation of the line tangent to the curve $x = 2 + 5\sec\theta$ and $y = 1 + 2\tan\theta$ at $\theta = \frac{\pi}{6}$

Sol/ to find $\frac{dy}{dx}$ we have $\frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{dy}{dx}$

$\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ then $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{dy}{dx}$

$\frac{dx}{d\theta} = \sec\theta \cdot \tan\theta$

and $\frac{dy}{d\theta} = 2\sec^2\theta \Rightarrow \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\sec^2\theta}{\sec\theta\tan\theta}$

$\frac{dy}{dx} = \frac{2\sec\theta}{\tan\theta}$

$= \frac{2 * \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{2}{\sin\theta} = 2\csc\theta$

$\therefore m = 2\csc\frac{\pi}{6} = 2 \times 2 = 4$

نعوض (θ) في (y,x) للحصول على النقطة

$x = 2 + \frac{1}{\cos\frac{\pi}{6}} \Rightarrow x = 2 + \frac{1}{\frac{\sqrt{3}}{2}} = 2 + \frac{2}{\sqrt{3}} = 3.154$

$y = 1 + 2\tan\frac{\pi}{6} = 1 + 2 \cdot \frac{1}{\sqrt{3}} = 1 + \frac{2}{\sqrt{3}} = \frac{\sqrt{3} + 2}{\sqrt{3}}$

$y = (2.154)$

$\therefore$ The point $(3.154, 2.154)$

$y - y_0 = m(x - x_0) \Rightarrow y - 2.154 = 4(x - 3.154)$

$y - 2.154 = 4x - 4 * 3.154$

$\Rightarrow y - 4x + 10.362 = 0$ equation