Cloud Physics Lab

LAB 10: Growth of Ice Crystals I

Diffusional Growth of Ice Crystals

Introduction:

Ice crystals can form inside cloud either by the freezing of super-cooled water droplets, or via deposition. Just as condensation requires a nucleus to provide a surface for the embryonic water droplet to form around, so too does deposition. Ice crystals must form around some nuclei. In this lab, student will explore the growth of ice crystal by diffusional process.

Objective:

a) Plot and study the diffusional growth of ice crystal for different temperatures.
b) Plot and study distance that ice crystal needs to fall inside the cloud for given masses at different temperatures.

Theory:

Diffusional growth of ice crystals is more rapid than that of liquid water droplets because the saturation vapor pressure over ice is less than that over water. The diffusional growth equation for ice crystals is very similar to that for water droplets,

$$\frac{dm}{dt} = \frac{4\pi C(S_i - 1)}{F_k + F_d} \tag{1}$$

where m is the mass of the ice crystal and C is the shape factor. The thermodynamics term (F_k) and the diffusion term (F_d) are given by:

$$F_k = \left(\frac{L_s}{R_v T} - 1\right)\frac{L_s}{KT} \tag{2}$$

$$F_d = \frac{R_v T}{D e_{si}(t)} \tag{3}$$

where

L_s is the latent heat of sublimation, ρ_l is the water density, R_v is the water vapor gas constant, K is thermal conductivity of air, T is the temperature, D is diffusivity of water vapor, and e_{si} is that saturation vapor pressure over ice, and not liquid water.

For a plate shape ice crystal, the shape factor C is $2R/\pi$. For the plate, R refers to the nominal radius of the equivalent disk. Substituting C into equation (1) gives:

$$\frac{dm}{dt} = \frac{8R(S_i - 1)}{F_k + F_d} \tag{4}$$

The mass of the equivalent disk is to $m = \pi R^2 h \rho_i$ where *h* is the thickness of the disk and ρ_i is the ice density. Substituting for *R* into equation (4) gives:

$$\frac{dm}{\sqrt{m}} = \frac{8}{\sqrt{\pi h \rho_i}} \frac{(S_i - 1)}{F_k + F_d} dt \qquad (5)$$

Equation (5) can be solved analytically if *h* is considered constant.
Using the chain rule:

$$\frac{dm}{dz} = \frac{dm}{dt}\frac{dt}{dz} = \frac{1}{u}\frac{dm}{dt} \qquad (6)$$

From equation (4) and (6) we get:

$$\frac{dm}{dz} = \frac{1}{u}\frac{8R(S_i - 1)}{F_k + F_d} \qquad (7)$$

If diameter and fall speed if the ice crystal are related by $u = xD$, where $x = 520\ s^{-1}$
Then equation (7) becomes:

$$\frac{dm}{dz} = \frac{1}{u}\frac{8R(S_i - 1)}{F_k + F_d} = \frac{1}{xD}\frac{4D(S_i - 1)}{F_k + F_d} = \frac{4}{x}\frac{(S_i - 1)}{F_k + F_d} \qquad (8)$$

where $x = 520\ s^{-1}$.

Materials and Procedures:

1. Run the Matlab script **Lab10a.m** diffusional growth of ice crystal for different temperatures.
2. Run the Matlab script **Lab10b.m** to plot the fall distance of ice crystal inside the cloud needed to grow to a given mass.

Analysis and Conclusions:

1. Use figure 1 to describe how temperature can affect the diffusional growth of ice crystal. Note that the maximum growth occurs at -15 ^{o}C.
2. Use figure 2 to describe how long ice crystal has to fall so that it can grow by diffusion to a given mass.

Questions:

1. What did you learn about the diffusional growth of ice crystal by completing this Lab?
2. In fig. 1, explain why the maximum diffusional growth rate of ice crystals occur at 15 ^{o}C. Also, explain why does the growth rate at -30 ^{o}C is greater than that at -5 ^{o}C?
3. Fig. 2 shows that for a given temperature the mass of an ice crystal increases linearly with fall distance. Why?