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Logic Design

Karnaugh Map

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Lecture Outlines

- What is K-Map?
- Why needs K-Map?
- How to represent K-Map?
  - 2-vartiables function
  - 3-variables function - example
  - 4-variables function -example
  - 5-variables function - example
- Simplify expressions using K-Maps
  - Grouping
- Some Examples
- Quiz

What is K-Map?

- Karnaugh map or shortly K-Map, is a two dimensional graphical representation technique used to simplify the Boolean algebra expressions or from truth tables

- It can be used to written minimal boolean expressions representing the required logic.

Why needs K-Map?

- Simplify using boolean algebra is more complex than using K-Map
- The result expression is perfectly simplified
- Working within SOP and POS
- Unknown truth table case(s) can be considered as don’t care (in Next Lecture)

How to represent K-Map?

- 2-Variables function

- A two-variable function has four possible minterms. We can re-arrange these minterms into a Karnaugh map.

x	y	minterm
0	0	$x'y'$
0	1	$x'y$
1	0	xy'
1	1	xy

		y	
		0	1
X	0	$x'y'$	$x'y$
	1	xy'	xy

- Now we can easily see which minterms contain common literals.
  - Minterms on the left and right sides contain y' and y respectively.
  - Minterms in the top and bottom rows contain x' and x respectively

		y	
		0	1
X	0	$x'y'$	$x'y$
	1	xy'	xy

	y'	y
X'	$x'y'$	$x'y$
X	xy'	xy

How to represent K-Map?

- 3-Variables function
  - For a three-variable expression with inputs x, y, z, the arrangement of minterms is more tricky:

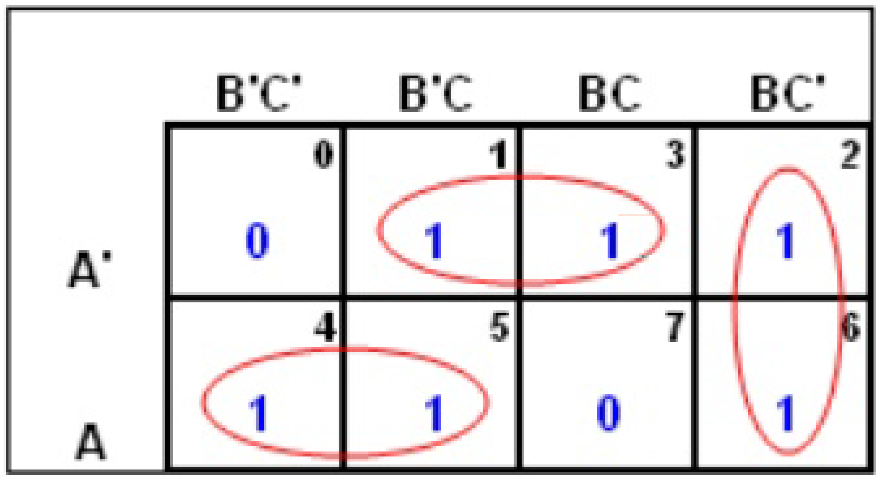
			y	
	x'y'z'	x'y'z	x'yz	x'yz'
x	xy'z'	xy'z	xyz	xyz'
		z		

	yz	00	01	11	10
x	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

How to represent K-Map?

- Example of three variables K-Map: given

$F(a,b,c) = \sum m\,(1, 2, 3, 4, 5, 6)$



$F = A'C + BC' + AB'$

How to represent K-Map?

- 4-variables function: F(W,X,Y,Z)
  - Grouping minterms is similar to the three-variable case, but:
    We can have rectangular groups of 1, 2, 4, 8 or 16 minterms.

			y	y	
	w'x'y'z'	w'x'y'z	w'x'yz	w'x'yz'	
	w'xy'z'	w'xy'z	w'xyz	w'xyz'	x
w	wxy'z'	wxy'z	wxyz	wxyz'	x
w	wx'y'z'	wx'y'z	wx'yz	wx'yz'	
		z	z		

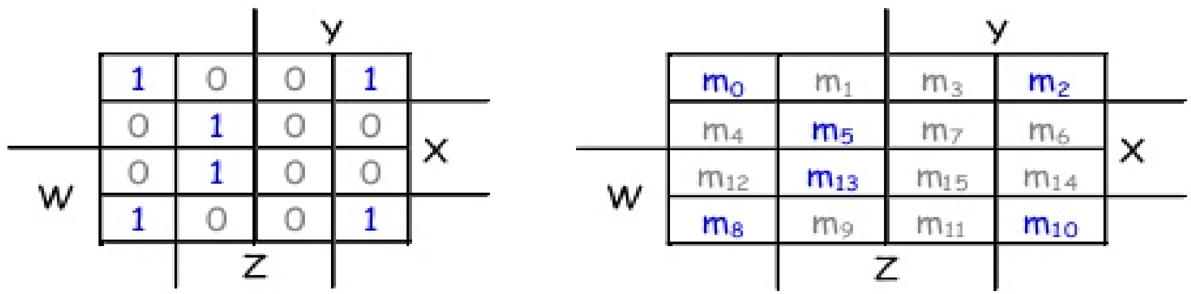
			y	y	
	m_0	m_1	m_3	m_2	
	m_4	m_5	m_7	m_6	x
w	m_{12}	m_{13}	m_{15}	m_{14}	x
w	m_8	m_9	m_{11}	m_{10}	
		z	z		

How to represent K-Map?

- Example: simplify the following (SOMs):

  $F(w,x,y,z)=\Sigma(m_0, m_2, m_5, m_8, m_{10}, m_{13})$

  - The expression is already a sum of minterms, so here's the K-map:



Result, $F=x'z' + xy'z$.

How to represent K-Map?

- 5-variables function
  - The #of locations needed is 2^n , n=#of variables
  - 2^5 =2x2x2x2x2=32 locations
  - We need 2x16 K-Maps for represents these 5-var.

F (A, B, C , D, E) = Σm (0, 2, 3, 5, 7, 8, 11, 13, 17, 19, 23, 24, 29, 30)

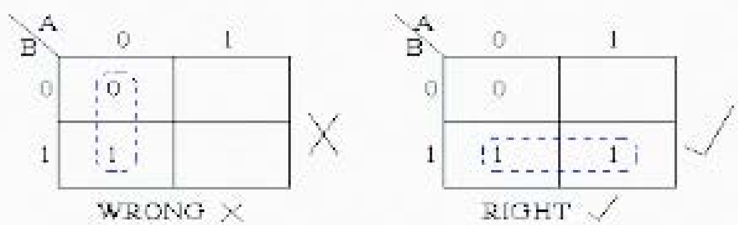
	A'				A			
	D'E'	D'E	DE	DE'	D'E'	D'E	DE	DE'
B'C'	0 (0)	0 (1)	1 (3)	1 (2)	0 (16)	1 (17)	1 (19)	0 (18)
B'C	0 (4)	1 (5)	1 (7)	0 (6)	0 (20)	0 (21)	1 (23)	0 (22)
BC	0 (12)	1 (13)	0 (15)	0 (14)	0 (28)	1 (29)	0 (31)	1 (30)
BC'	1 (8)	0 (9)	1 (11)	0 (10)	1 (24)	0 (25)	0 (27)	0 (26)

F = B'DE + A'C'DE + A'B'C'E' + A'B'CE + AB'C'E + BCD'E + BC'D'E' + ABCDE'

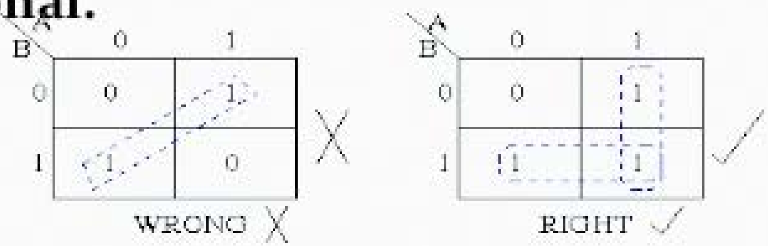
Simplify expressions using K-Maps

- K-Map uses the following rules for simplifying expressions by grouping the cells containing ONES only.

1. Groups may not include any cell containing a zero.

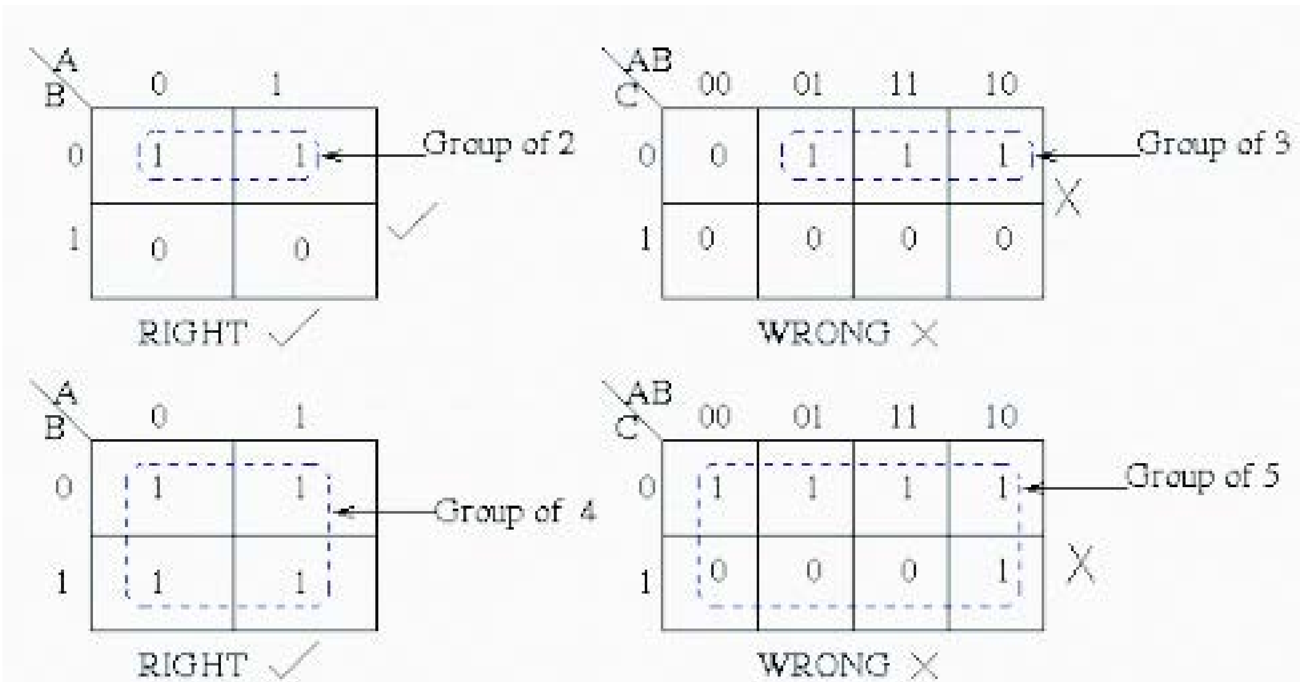


2. Groups may be horizontal or vertical, but not diagonal.



Simplify expressions using K-Maps

- Groups made using 2^n cells only.
  - If n=1, a group contains two 1's since $2^1=2$
  - If n=2, a group contains four 1's since $2^2=4$



Simplify expressions using K-Maps

4. Each group should be as large as possible.

C \ AB	00	01	11	10
0	1	1	1	1
1	0	0	1	1

RIGHT ✓

C \ AB	00	01	11	10
0	1	1	1	1
1	0	0	1	1

WRONG ×

(Note that no Boolean laws broken, but not sufficiently minimal)

5.Each cell containing a one must be in at least one group.

C \ AB	00	01	11	10
0	0	0	1	1
1	0	0	0	1

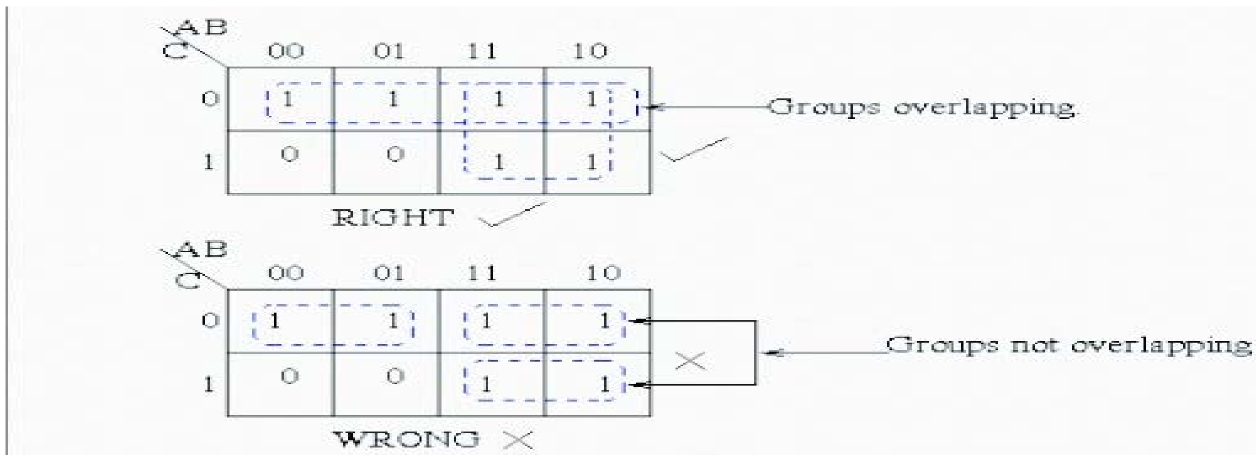
Group I

1 present in at least one group

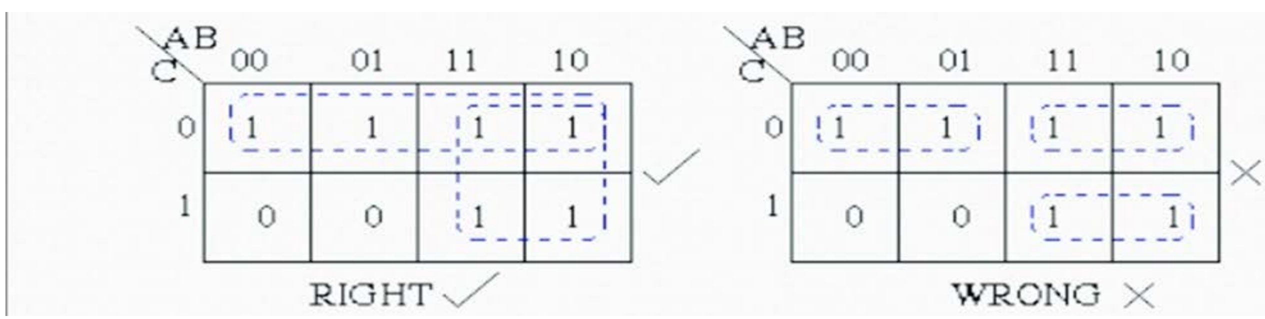
Group II

Simplify expressions using K-Maps

6. Groups may Overlap.

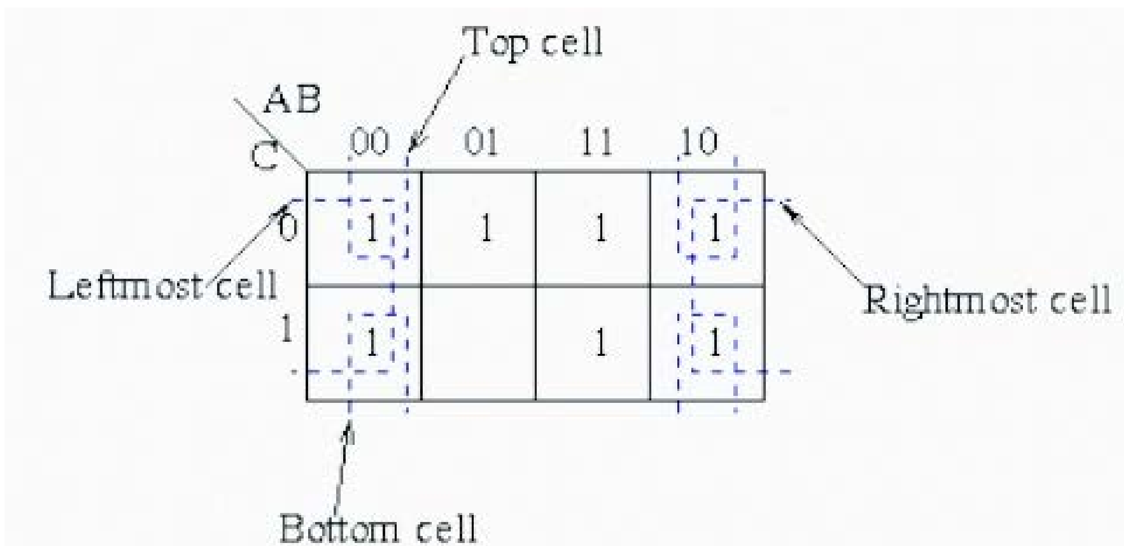


7. there should be as few groups as possible.



Simplify expressions using K-Maps

8. groups may wrap around the table.
   - The leftmost cell in a row may be grouped with the rightmost cell and the top cell in a column may be grouped with the bottom cell.



Examples:

- Simplify the following expression using: (a) boolean algebra, (b) K-Map.

$$F(x, y)=x+xy$$

Sol.

a. Using boolean algebra

$F=X+XY \rightarrow F=X(1+Y)$ since (1+Y)=1 in Boolean Rules

$\rightarrow F=X$

b. Using K-Map: we need first to get Minterms:

Express the function in SOMs:

a. $F=x+xy \rightarrow F=x.1+xy$

$\rightarrow F=x(y+y')+xy$

$\rightarrow F=xy+xy'+xy$ since $xy+xy=xy$,

$\rightarrow F=xy'+xy \rightarrow F(x, y)=\Sigma(m_2, m_3)$

Simplified Function $F(x, y)=x$

b. Using Truth Table

	y'	y
x'		
x	1	1

x	y	F
0	0	0
0	1	0
1	0	1
1	1	1

$F(x, y)=x$

Examples:

- Simplify the following expression using K-Map in a. SOP, b. POS

$$F(A,B,C,D)=\Sigma(0, 2, 3, 6, 8, 9, 10, 12)$$

Sol.

$F=A'B'C'D'+A'B'CD'+A'B'CD+A'BCD'+AB'C'D'+AB'C'D+AB'CD'+ABC'D'$

	C'D'	C'D	CD	CD'
A'B'	1	0	1	1
A'B	0	0	0	1
AB	1	0	0	0
AB'	1	1	0	1

SOP:
F=

POS:
F'=

F=

Quiz

- Try yourself to solve this question:
  - Simplify the following function's truth table using K-Map in SOP and POS, then draw the SOP circuit?

x	y	z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

End of this Lecture!

Any Questions?

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class join code: upoi4fe