

HEAP SORT

INTRODUCTION

In heap sort, the given elements are first arranged in a heap. Then the elements are removed one by one. After every deletion, the elements are re-heapified. If the heap in question is a max-heap, then we get the elements in the descending order. In the case of a min-heap, the output is an ascending sequence.

The following algorithm presents the formal procedure of heapsort.

Algorithm 6.3 Heapsort

Input: A list of elements

Output: Sorted elements

Strategy: Discussed above

Heapsort (List elements) returns sorted_list

```
{
    heap h=heapify (elements);
//the elements are heapified and inserted into a heap namely h
    i=0;
    while(i!=n)
        {
            x=delete(
//the delete function removes the element at the root of the heap and inserts
it into x
                insert(sorted_list, x);
//the element x is inserted into sorted_list
            }
}
```

Complexity: As explained earlier, the algorithm requires heapify. The worst-case complexity of heapify is $(\log(n))$; therefore, the complexity of the algorithm is $O(n \log n)$.

Analysis of Heap sort Algorithm:

- In-place sorting algorithm – memory efficient
- Time complexity – $O(n \log(n))$
- 1st Step- Build heap, $O(n)$ time complexity
- 2nd Step – perform n delete Max operations, each with $O(\log(n))$ time complexity

Total time complexity = $O(n \log(n))$

- Fast sorting algorithm, memory efficient, especially for very large values of n .
- **Slower of the $O(n \log(n))$ sorting algorithms**

The binary heap data structure is an array that can be viewed as a complete binary tree. Each node of the binary tree corresponds to an element of the array. The array is completely filled on all levels except possibly lowest.

An Array A that presents a heap with two attribute:

- Length $[A]$: the number of elements in the array.
- heap- size $[A]$: the number of elements in the heap stored with array A .
- Length $[A] \geq$ heap-size $[A]$.

To convert the heap tree into a heap array as shown in the figure below , the root of the tree $A[0]$ and given index i of a node , the indices of its parent , left child and right child can be computed as :

- $A[0]$ is the root of the tree
- The PARENT (i) is at $[(i-1)/2]$ if $i \neq 0$.
- The left child LEFT (i) is at $[2i+1]$
- The right child RIGHT(i) is at $[2i+2]$

Complete Binary Trees , and Heap

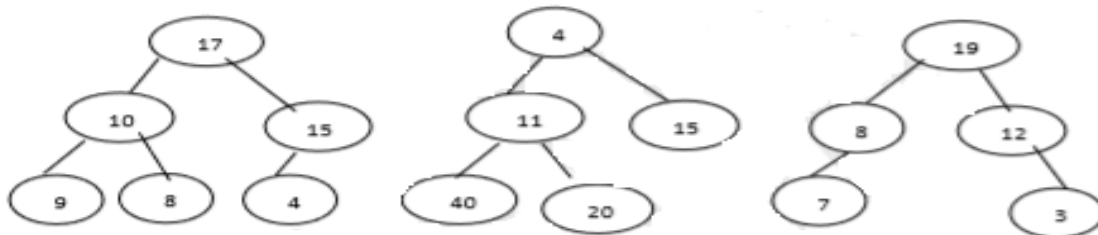


Array representation:

0	1	2	3	4	5	6	7	8	9
50	24	30	20	21	18	3	12	5	6

Heap Sort is the one of the most efficient comparison-based algorithms, is a sorting algorithm that works by first organizing the data to be sorted into a binary heap data structure (uses a heap as its data structure).

Example: which of the following trees is max-heap, min-heap ,non heap



Max-heap

min-heap

non-heap

Step by step Example:

The heap sort algorithm consists of procedure :

BuildHeap (A)

Build a heap out of array A , the procedure BuildHeap goes through the remaining nodes of the tree and runs MaxHeapify procedure on each one.

MaxHeapify(A,i) or MinHeapify(A,i)

Make the sub tree of A starting in node i fulfill the heap property (MaxHeapify picks the largest child and compare it to the parent . If parent is larger than MaxHeapify quits , otherwise it swaps the parent with the largest child . So that the parent is now becomes larger than its children).

HeapSort(A)

Make A heap , then take out the root , repeat until the array is sorted.

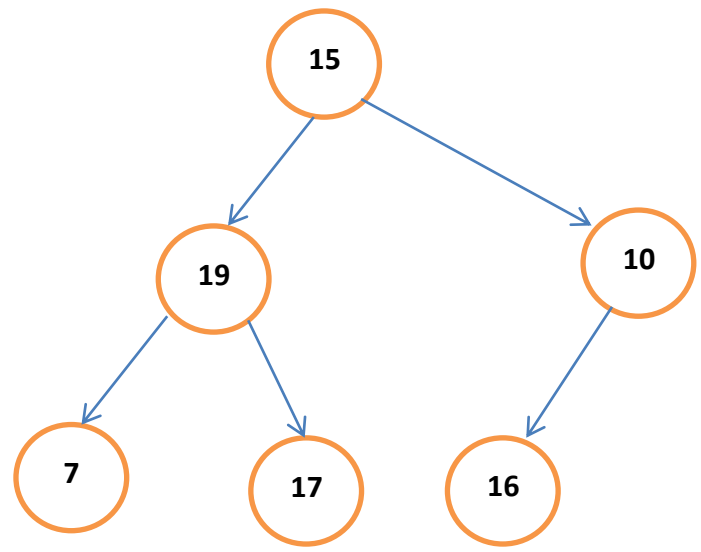
Example:

Here is the array: 15, 19, 10, 7, 17, 6

Building the heap tree

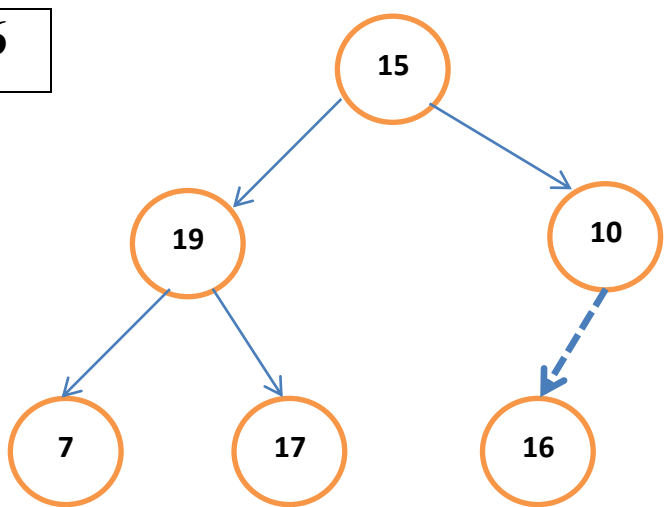
The array represented as a tree, complete but not ordered.

15	19	10	7	17	16
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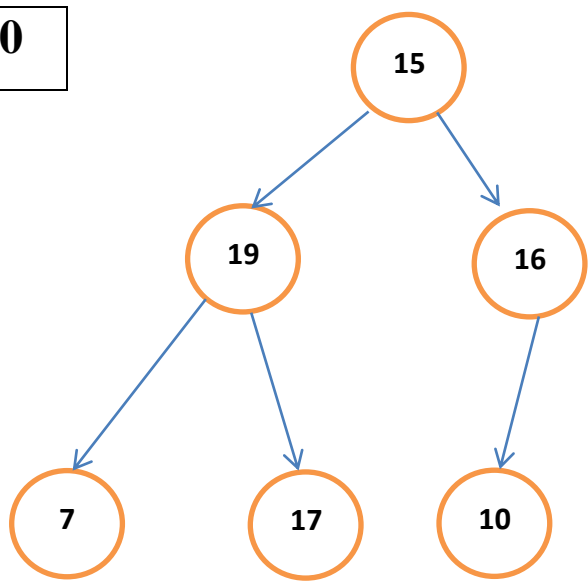
Start with the rightmost node at height 1-the node at position $3 = \text{size}/2$. It has one greater child and has to be percolated down:

15	19	10	7	17	16
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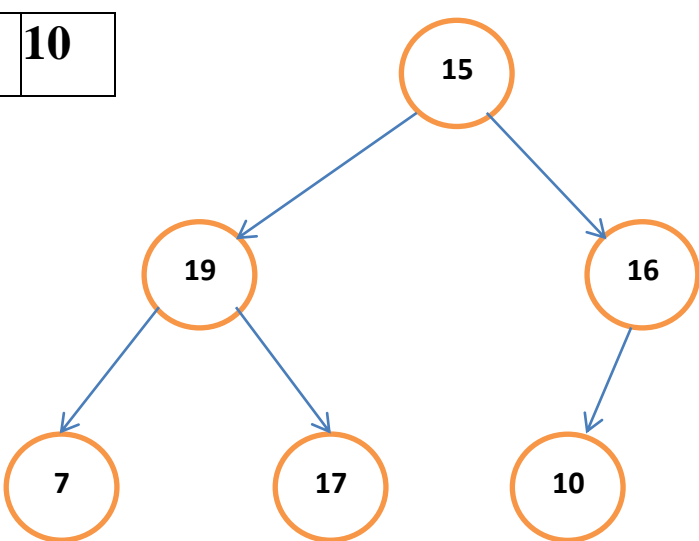
After processing array [3] the situation is:

15	19	16	7	17	10
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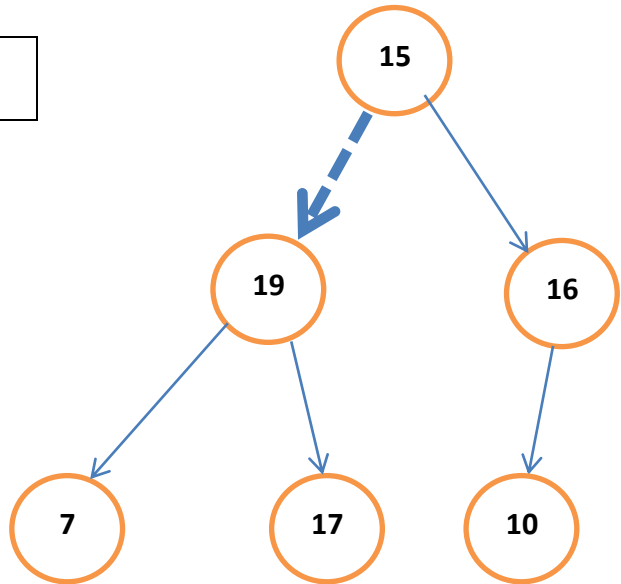
Next comes array [2]. Its children are smaller, so no percolation is needed.

15	19	16	7	17	10
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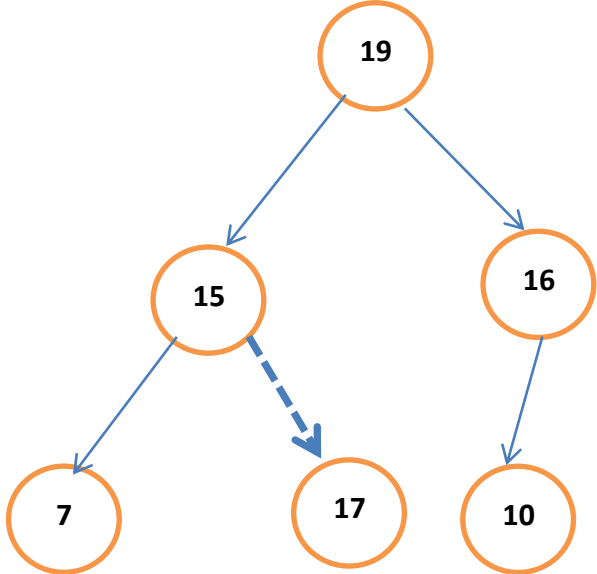
The last node be processed is array [1]. Its left child is the grater of the children. The item at array [1] has to be percolated down to the left, swapped with array [2].

15	19	16	7	17	10
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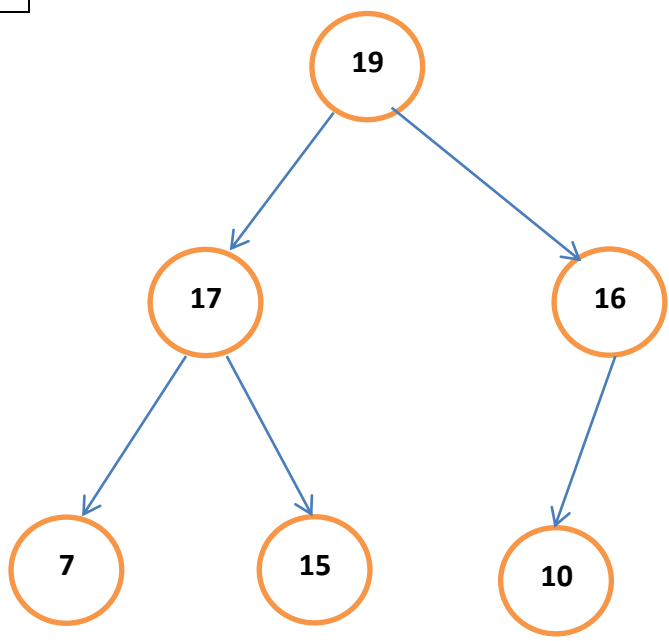
As a result the situation is:

19	15	16	7	17	10
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The children of array [2] are greater, and item 15 has to be moved down further, swapped with array [5].

19	17	16	7	15	10
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Now the tree is ordered, and the binary heap is built.