Chapter -5- System of Numbers-

Consider the following elements :-

ϕ

$\{\phi\}$

$\{\phi, \{\phi\}\}$

$\{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$

$\vdots$

and we introduce the usual numerical symbols $0, 1, 2, 3, \ldots$ for these various sets. The symbol $n = 0, 1, 2, 3, \ldots$ therefore represents the set above with n elements.

It follows from definition that

$0 = \{\phi\}$

$1 = \{0\}$

$2 = \{0, 1\}$

$3 = \{0, 1, 2\}$

$\vdots$

Definition

the successor of x is $x \cup \{x\}$ and is denoted by x^+

Thus,

$0^+ = 0 \cup \{0\}$

$= \phi \cup \{\phi\}$

$= \{\phi\} = 1$

$1^+ = 1 \cup \{1\}$

$= \{\phi\} \cup \{\{\phi\}\}$

$= \{\phi, \{\phi\}\} = 2 = \{0, 1\}$

$2^+ = 2 \cup \{2\}$

$= \{\phi, \{\phi\}\} \cup \{\{\phi, \{\phi\}\}\}$

$= \{\phi, \{\phi\}, \{\{\phi, \{\phi\}\}\}\}$

$= 3$

$\vdots$

$n^+ = n+1$

Definition

Let A be a set, we call A a successor set if

① $0 \in A$

② $x^+ = x \cup \{x\} \in A$ whenever $x \in A$.

Notes :-

مطلوب برهان ① Any successor set should contains the numbers $0, 1, 2, \ldots, n$ ✗

مطلوب برهان ② Collection of all successor sets is not empty ✗

③ Intersection of any non empty collection of successor sets is also successor set.

Definition :-

Intersection of all successor sets is called the set of natural numbers and is denoted by $\mathbb{N}$.

Each element of $\mathbb{N}$ is called natural element.

Peano Postulates :-

مطلوب برهان ① $0 \in \mathbb{N}$

② If $a \in \mathbb{N}$, then $a^+ \in \mathbb{N}$

مطلوب ③ $0 \neq a^+ \in \mathbb{N}$ for every natural number a

مطلوب ④ If $a^+ = b^+$, then $a = b$ for any natural numbers a, b.

مطلوب ⑤ If X is a successor subset of $\mathbb{N}$, then $X = \mathbb{N}$

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The intersection of any non-empty collection of successor sets is a successor set.

Let $B = \bigcap(\text{successor sets})$

$\therefore$ $0 \in B$ since $0 \in$ each successor set.

Let $z \in B$, then $z \in \bigcap(\text{Successor sets})$ and $z \in$ every successor set

then $z^+ \in$ every successor set and $z^+ \in \bigcap(\text{Successor sets})$

then $z \in B^+$ and B is a successor set.

Peano Posulates

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Clear from Theorem (*), below.

Theorem ⊗

There is a smallest successor set.
Means there is a successor set which is a subset of all successor sets. This smallest successor set is of course the intersection of all successor sets.

Proof

The intersection of all successor sets is a successor set.

Now To prove that the intersection is the smallest successor set, we need only to prove that the intersection is a subset of every successor set.

let $S := \bigcap$ all successor sets

and let $x \in S$, and let t by any successor set

$\{x, t\}$ is a set, and $x \cap t$ is a set and it is a successor set

$S \subseteq x \cap t$

Hence, S is the smallest successor set.