# Derivative المشتقة

## Rules of Derivatives: قواعد في المشتقة

1. $\frac{d}{dx}(c) = 0$ c is any number
2. $\frac{d}{dx}(x^n) = n x^{n-1}$
3. $\frac{d}{dx}(C * F(x)) = C * \frac{d}{dx} F(x)$
4. $\frac{d}{dx}(F(x) \mp g(x)) = \frac{d}{dx} F(x) \mp \frac{d}{dx} g(x)$
5. $\frac{d}{dx}\left(\frac{F(x)}{g(x)}\right) = \frac{g(x) * \frac{d}{dx} F(x) - F(x) * \frac{d}{dx} g(x)}{(g(x))^2}$
6. $\frac{d}{dx}(F(x) \cdot g(x)) = F(x) \cdot \frac{d}{dx} g(x) + g(x) * \frac{d}{dx} F(x)$
7. $\frac{d}{dx}(F(x))^n = n(F(x))^{n-1} * \frac{d}{dx} F(x)$
8. $\frac{d}{dx}(\sin u) = \cos u * \frac{du}{dx}$ مشتقة الزاوية
9. $\frac{d}{dx}(\cos u) = -\sin u * \frac{du}{dx}$
10. $\frac{d}{dx}(\tan u) = \sec^2 u * \frac{du}{dx}$
11. $\frac{d}{dx}(\cot u) = -\csc^2 u * \frac{du}{dx}$
12. $\frac{d}{dx}(\sec u) = \sec u * \tan u \frac{du}{dx}$
13. $\frac{d}{dx}(\csc u) = -\csc u * \cot u * \frac{du}{dx}$
14. $\frac{d}{dx}\sqrt{F(x)} = \frac{\text{مشتقة داخل الجذر}}{2 * \sqrt{\text{الجذر نفسه}}}$

1

**ex ①** Find $\frac{dy}{dx}$ , if $y = 2x^4 + 5x^2 + x + 16$

**Sol** $\frac{dy}{dx} = 8x^3 + 10x + 1$

**ex ②** $y = x \cdot \cos^2(x^3+1)$ find $\frac{dy}{dx}$

**Sol** $\frac{dy}{dx} = x * 2\cos(x^3+1) * -\sin(x^3+1) * 3x^2$

$+ \cos^2(x^3+1) * 1$ (مسألة رقم 6)

**ex ③** $y = \cot\left(\frac{x}{x+1}\right)$ find $\frac{dy}{dx}$

**Sol** $\frac{dy}{dx} = -\csc^2\left(\frac{x}{x+1}\right) * \left[\frac{(x+1)*1 - x*1}{(x+1)^2}\right]$

$= -\csc^2\left(\frac{x}{x+1}\right) * \left[\frac{\cancel{x}+1-\cancel{x}}{(x+1)^2}\right]$

$\therefore \frac{dy}{dx} = -\csc^2\left(\frac{x}{x+1}\right) * \left(\frac{1}{(x+1)^2}\right)$

**ex ④** $y = x \cdot \cos x + \tan x^2$ find $\frac{dy}{dx}$

**Sol**

$\frac{dy}{dx} = x * -\sin x + \cos x * 1 + \sec^2 x^2 * 2x$

$\therefore \frac{dy}{dx} = -x\sin x + \cos x + 2x\sec^2 x^2$

ex ⑤ $y = \sin^2 x^2 - \cos^3(3x) + 1$

Sol $\frac{dy}{dx} = 2\sin x^2 \cdot \cos x^2 \cdot 2x - 3\cos^2(3x) - \sin 3x * 3 + 0$

$= 4x \cdot \sin x^2 \cos x^2 + 9\cos^2(3x) \sin 3x.$

ex ⑥ $y = (x^2+1)^5$ find $\frac{dy}{dx}$

Sol $\frac{dy}{dx} = 5(x^2+1)^4 * 2x$

$= 10x (x^2+1)^4$

ex ⑦ $y = [(5-x)(4-2x)]^2$ find $\frac{dy}{dx}$

Sol $\frac{dy}{dx} = 2[(5-x)(4-2x)] * [(5-x)*(-2) + (4-2x)* -1]$

ex (8): $y = (2x^3 - 3x^2 + 6x)^{-4}$

Sol $\frac{dy}{dx} = -4(2x^3 - 3x^2 + 6x)^{-5} * (6x^2 - 6x + 6)$

REDMI NOTE 8 PRO
64MP QUAD CAMERA

Ex 9 $\quad y = \frac{12}{x} + \frac{4}{x^3} - \frac{3}{x^4}$

Sol $\quad \frac{dy}{dx} = \frac{x*0-12*1}{x^2} + \frac{x^3*0-4*3x^2}{x^6} - \frac{x^4*0-3*4x^3}{x^8}$

$\therefore \frac{dy}{dx} = \frac{-12}{x^2} - \frac{12x^2}{x^6} + \frac{12x^3}{x^8}$

$= \frac{-12}{x^2} - \frac{12}{x^4} + \frac{12}{x^5} = \frac{12}{x^5} - \frac{12}{x^2} - \frac{12}{x^4}$

ex 10 $\quad y = \frac{(x^2+x)(x^2-x+1)}{x^4}$

Sol $\quad \frac{dy}{dx} = \frac{x(x+1)(x^2-x+1)}{x^4} = \frac{(x+1)(x^2-x+1)}{x^3}$

$\therefore \frac{dy}{dx} = \frac{x^3*[(x+1)*(2x-1)+(x^2-x+1)*1]-(x+1)(x^2-x+1)*3x^2}{(x^3)^2}$

$\therefore \frac{dy}{dx} = \frac{x^2[(x+1)(2x-1)+(x^2-x+1)]-3(x+1)(x^2-x+1)}{\cancel{x^6}}$

$\therefore \frac{dy}{dx} = \frac{(x+1)(2x-1)+(x^2-x+1)-3(x+1)(x^2-x+1)}{x^4}$

ex 11 $y = \tan(\sec x)$ find $y'$

Sol $\frac{dy}{dx} = \sec^2(\sec x) * \sec x * \tan x$.

ex 12 $y = \tan^3\left(\frac{x+1}{x-1}\right)$

Sol $\frac{dy}{dx} = 3\tan^2\left(\frac{x+1}{x-1}\right) * \sec^2\left(\frac{x+1}{x-1}\right) * \left[\frac{(\cancel{x}-1)*1-(\cancel{x}+1)*1}{(x-1)^2}\right]$

$= 3\tan^2\left(\frac{x+1}{x-1}\right) * \sec^2\left(\frac{x+1}{x-1}\right) * \left[\frac{-2}{(x-1)^2}\right]$

$= \frac{-6}{(x-1)^2} \cdot \tan^2\left(\frac{x+1}{x-1}\right) \cdot \sec^2\left(\frac{x+1}{x-1}\right)$

ex 13 $y = \tan^{\frac{1}{2}}\sqrt{2x+7}$

Sol $\frac{dy}{dx} = \frac{1}{2}\tan^{-\frac{1}{2}}\sqrt{2x+7} \cdot \sec^2\sqrt{2x+7} * \frac{\cancel{2}}{\cancel{2}\sqrt{2x+7}}$

$\therefore \frac{dy}{dx} = \frac{\sec^2\sqrt{2x+7}}{2\tan\sqrt{2x+7} \cdot \sqrt{2x+7}}$

REDMI NOTE 8 PRO
64MP QUAD CAMERA

ex 14

$y = x^2 \sin x$

Sol

$$\frac{dy}{dx} = x^2 \cos x + \sin x * 2x$$

$$= x^2 \cos x + 2x \sin x.$$

ex 15 $\quad y = \csc^{-\frac{2}{3}} \sqrt{5x}$ find $y'$

Sol

$$\frac{dy}{dx} = -\frac{2}{3} * \csc^{-\frac{5}{3}} \sqrt{5x} * -\csc\sqrt{5x} * \cot\sqrt{5x} * \frac{5}{2\sqrt{5x}}$$

$$\therefore \frac{dy}{dx} = \frac{5}{3\sqrt{5x}} \cdot \csc^{-\frac{2}{3}} \sqrt{5x} \cdot \cot\sqrt{5x}.$$

[illegible]

$\csc$ [illegible] $\leftarrow \boxed{-\frac{5}{3}+1=-\frac{2}{3}}$

# Higher derivative المشتقات العليا

وهو الاشتقاق لعدد من المرات للدالة :-

ex ①

find $\frac{d^4y}{dx^4}$ if $f(x) = 3x^4 + 2x - 6x^2 - 4$

Sol

$$\frac{dy}{dx} = 12x^3 + 2 - 12x$$

$$\frac{d^2y}{dx^2} = 36x^2 - 12$$

$$\frac{d^3y}{dx^3} = 72x$$

$$\frac{d^4y}{dx^4} = 72$$

ex ②

find The Third derivative of the function

$$y = \sqrt{x^3}$$

Sol

$$y = x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} \Rightarrow \frac{d^2y}{dx^2} = \frac{3}{2} * \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{3}{4}x^{-\frac{1}{2}}$$

$$\frac{d^3y}{dx^3} = \frac{3}{4} * -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{3}{8} * x^{-\frac{3}{2}}$$

$$= -\frac{3}{8} * \frac{1}{\sqrt{x^3}}$$

$$\therefore \frac{d^3y}{dx^3} = -\frac{3}{8y}$$

$\boxed{y = x^{\frac{3}{2}} \text{ ليش}}$

ex ③: Find The Second derivate for The

$$F(x) = \sqrt{2x} + \frac{2\sqrt{2}}{\sqrt{x}} \quad \text{at } \boxed{x=2}$$

**Sol**

$$\frac{dy}{dx} = \frac{\cancel{2}}{\cancel{2}\sqrt{2x}} + \cancel{2}\sqrt{2} * -\frac{1}{\cancel{2}} x^{-\frac{3}{2}}$$

$$= (2x)^{-\frac{1}{2}} + (-\sqrt{2}) \cdot x^{-\frac{3}{2}} = (2x)^{-\frac{1}{2}} - \sqrt{2}\, x^{-\frac{3}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{\cancel{2}}(2x)^{-\frac{3}{2}} * \cancel{2} - \sqrt{\cancel{2}} * -\frac{3}{\cancel{2}_{\sqrt{2}}} \cdot x^{-\frac{5}{2}}$$

$$= -(2x)^{-\frac{3}{2}} + \frac{3}{\sqrt{2}} x^{-\frac{5}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{3}{\sqrt{2} * \sqrt{x^5}} - \frac{1}{(2x)^{3/2}}$$

الآن نعوض عن قيمة $\boxed{x=2}$

$$= \frac{3}{\sqrt{2} * \sqrt{(2)^5}} - \frac{1}{(2*2)^{3/2}} = \frac{3}{\sqrt{2} * \sqrt{32}} - \frac{1}{(2^2)^{3/2}}$$

$$= \frac{3}{\sqrt{2} * 4\sqrt{2}} - \frac{1}{2^3} = \frac{3}{4*2} - \frac{1}{8}$$

$$= \frac{3}{8} - \frac{1}{8} = \frac{2}{8} = \boxed{\frac{1}{4}}$$

REDMI NOTE 8 PRO
64MP QUAD CAMERA

ex ④ Find $\frac{d^2y}{dx^2}$ For $y=\sqrt{u}+2u$

when $u=x^2-3$ at $x=2$

Sol

$$y=\sqrt{x^2-3}+2(x^2-3)$$

$$y=\sqrt{x^2-3}+2x^2-6$$

$$\frac{dy}{dx}=\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-3}}+4x=\frac{x}{(x^2-3)^{\frac{1}{2}}}+4x$$

$$\frac{d^2y}{dx^2}=\frac{(x^2-3)^{\frac{1}{2}}*1-x*\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-3}}}{\left[(x^2-3)^{\frac{1}{2}}\right]^2}+4$$

$$=\frac{\sqrt{x^2-3}-\frac{x^2}{\sqrt{x^2-3}}}{x^2-3}+4=\frac{\frac{\cancel{x^2}-3-\cancel{x^2}}{\sqrt{x^2-3}}}{x^2-3}+4$$

$$=\frac{-3}{(x^2-3)(x^2-3)^{\frac{1}{2}}}+4$$

$$=\frac{-3}{(x^2-3)^{\frac{3}{2}}}+4$$

عندما نعوض عن $x=2$ في المعادلة ينتج

$$=\frac{-3}{(4-3)^{3/2}}+4$$

$$=\frac{-3}{1}+4=\boxed{1}$$

**Ex 5** show that if $y = 35x^4 - 30x^2 - 3$

prove that $(1-x^2)y'' - 2xy' + 20y = 0$

**Sol**

$$y' = 4 * 35x^3 - 2 * 30x + 0$$

$$= 140x^3 - 60x$$

$$y'' = 140 * 3x^2 - 60 * 1 = 420x^2 - 60$$

الطرف الأيمن $= (1-x^2) * (420x^2 - 60) - 2x(140x^3 - 60x) + 20y$

$$= (1-x^2)(420x^2 - 60) - 2x(140x^3 - 60x) + 20(35x^4 - 30x^2 + 3)$$

$$= 420x^2 - 60 - 420x^4 + 60x^2 - 280x^4 + 120x^2 + 700x^4 - 600x^2 + 60$$

الطرف الأيسر $= 0$

∴ الطرف الأيمن = الطرف الأيسر $= 0$

# Implicit differentation (الاشتقاق الضمني)

ملحوظة :

يستخدم هذا النوع من الاشتقاق عند وجود $(x, y)$ في نفس الدالة سويًا.

**ex ①** Find $\frac{dy}{dx}$ for $xy + 2x - 5y = 2$

**Sol** $x * \frac{dy}{dx} + y * 1 + 2 - 5\frac{dy}{dx} = 0$

نستخرج $\frac{dy}{dx}$ عامل مشترك

$(x-5)\frac{dy}{dx} = -y-2$

$x\frac{dy}{dx} - 5\frac{dy}{dx} = -y - 2$

$\therefore \frac{dy}{dx} = \frac{-y-2}{x-5}$

**ex ②** find $\frac{dy}{dx}$ for $x^2y^2 = x^2 + y^2$

**Sol** $x^2 * 2y\frac{dy}{dx} + y^2 * 2x = 2x + 2y\frac{dy}{dx}$

$2yx^2\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - 2xy^2$

$\frac{dy}{dx}(2yx^2 - 2y) = 2x - 2xy^2$

$\frac{dy}{dx} = \frac{2x - 2xy^2}{2yx^2 - 2y} = \frac{\cancel{2}(x - xy^2)}{\cancel{2}(yx^2 - y)}$

$\therefore \frac{dy}{dx} = \frac{x - xy^2}{yx^2 - y}$

ex ③ find $\frac{dy}{dx}$ for $(x+y)^3+(x-y)^3=x^4+y^4$

at $A(2,1)$

Sol

$$3(x+y)^2\left[1+\frac{dy}{dx}\right]+3(x-y)^2*\left[1-\frac{dy}{dx}\right]$$
$$=4x^3+4y^3*\frac{dy}{dx}$$

$$3(x+y)^2+3(x+y)^2\frac{dy}{dx}+3(x-y)^2-3(x-y)^2\frac{dy}{dx}$$
$$=4x^3+4y^3\frac{dy}{dx}$$

$$3(x+y)^2\frac{dy}{dx}-3(x-y)^2\frac{dy}{dx}-4y^3\frac{dy}{dx}$$
$$=4x^3-3(y+y)^2-3(x-y)^2$$

$$\frac{dy}{dx}\left[3(x+y)^2-3(x-y)^2-4y^3\right]=4x^3-3(x+y)^2-3(x-y)^2$$

$$\therefore \frac{dy}{dx}=\frac{4x^3-3(x+y)^2-3(x-y)^2}{3(x+y)^2-3(x-y)^2-4y^3}$$

نعوض بالنقطة $A(2,1)$ في المعادلة

$$\frac{dy}{dx}=\frac{4*(2)^3-3(2+1)^2-3(2-1)^2}{3(2+1)^2-3(2-1)^2-4(1)^3}$$

$$\frac{dy}{dx}=\frac{32-3*9-3}{3*9-3-4}$$

$$\therefore \frac{dy}{dx}=\frac{32-30}{27-7}=\frac{2}{20}=\frac{1}{10}$$

ex 4

Find $\frac{dy}{dx}$ for $\sin xy = \tan^2 x^2 - \sin(x+y) + 3\pi$

Sol

$\cos xy * [x * \frac{dy}{dx} + y * 1] = 2\tan x^2 \cdot \sec^2 x^2 * 2x - \cos(x+y) * [1 + \frac{dy}{dx}]$

$x \cos xy \frac{dy}{dx} + y \cos xy = 4x \cdot \tan x^2 \cdot \sec^2 x^2 - \cos(x+y) - \cos(x+y)\frac{dy}{dx}$

$x \cos xy \frac{dy}{dx} + \cos(x+y)\frac{dy}{dx} = 4x \cdot \tan x^2 \cdot \sec^2 x^2 - \cos(x+y) - y\cos xy$

$\frac{dy}{dx}(x \cdot \cos xy + \cos(x+y)) = 4x \cdot \tan x^2 \cdot \sec^2 x^2 - \cos(x+y) - y \cos xy$

$$\frac{dy}{dx} = \frac{4x \tan x^2 \cdot \sec^2 x^2 - \cos(x+y) - y\cos xy}{x \cdot \cos xy + \cos(x+y)}$$

ex 5 find $\frac{dy}{dx}$ for $\sqrt{x \cdot y} + 1 = y$

Sol

$$\frac{x * \frac{dy}{dx} + y * 1}{2\sqrt{x \cdot y}} + 0 = \frac{dy}{dx}$$

$$\frac{x\frac{dy}{dx} + y}{\frac{dy}{dx}} = 2\sqrt{x \cdot y} \Rightarrow \frac{x\frac{dy}{dx}}{\frac{dy}{dx}} + \frac{y}{\frac{dy}{dx}} = 2\sqrt{x \cdot y}$$

$$x + \frac{y}{\frac{dy}{dx}} = 2\sqrt{x \cdot y} \Rightarrow \frac{y}{\frac{dy}{dx}} = 2\sqrt{x \cdot y} - x$$

$$\therefore \frac{dy}{dx} = \frac{y}{2\sqrt{x \cdot y} - x}$$

ex 6 find $\frac{dy}{dx}$ for $y^2 \sin(xy) = \tan x$.

Sol

$$y^2 * \cos(xy)\left[x * \frac{dy}{dx} + y * 1\right] + \sin(xy) * 2y\frac{dy}{dx} = \sec^2 x$$

$$xy^2 \cos(xy)\frac{dy}{dx} + y^3 \cos(xy) + 2y\sin(x \cdot y)\frac{dy}{dx} = \sec^2 x$$

$$xy^2 \cos(xy)\frac{dy}{dx} + 2y\sin(xy)\frac{dy}{dx} = \sec^2 x - y^3 \cdot \cos(xy)$$

$$\frac{dy}{dx}\left(xy^2 \cdot \cos(xy) + 2y\sin(xy)\right) = \sec^2 x - y^3 \cos(xy)$$

$$\therefore \frac{dy}{dx} = \frac{\sec^2 x - y^3 \cdot \cos(xy)}{xy^2 \cdot \cos(xy) + 2y\sin(xy)}$$

ex 7 find $\frac{dy}{dx}$ for $\frac{xy^3}{1+\sec y} = 1+x^4$

Sol

$$xy^3 = (1+\sec y)(1+x^4)$$

$$xy^3 = 1+x^4+\sec y + x^4 \sec y$$

$$x * 3y^2 \frac{dy}{dx} + y^3 * 1 = 4x^3 + \sec y . \tan y \frac{dy}{dx}$$
$$+ [x^4 * \sec y . \tan \frac{dy}{dx} + \sec y * 4x^3$$

$$3y^2 x \frac{dy}{dx} + y^3 = 4x^3 + \sec y . \tan y \frac{dy}{dx}$$
$$+ x^4 \sec y \tan y \frac{dy}{dx} + 4x^3 * \sec y$$

$$3y^2 x \frac{dy}{dx} - \sec y \tan y \frac{dy}{dx} - x^4 \sec y \tan y \frac{dy}{dx}$$
$$= 4x^3 + 4x^3 \cdot \sec y - y^3$$

$$\frac{dy}{dx}(3y^2 x - \sec y . \tan y - x^4 . \sec y . \tan y)$$
$$= 4x^3 + 4x^3 \cdot \sec y - y^3$$

$$\therefore \frac{dy}{dx} = \frac{4x^3 + 4x^3 \cdot \sec y - y^3}{3y^2 x - \sec y . \tan y - x^4 . \sec y \tan y}$$

# Chain Rules قانون السلسلة

ملاحظة :- توجد في هذا الموضوع قاعدتان وهما

① If $y = F(t)$ and $t = g(x)$ then

* نشتق كل منهما على حدة . والحصول على $\left(\frac{dy}{dx}\right)$ نجري

$$\frac{dy}{dx} = \frac{dy}{dt} * \frac{dt}{dx}$$

ex ① Find $\frac{dy}{dx}$ if $y = t^5 + 1$ and $t = \sqrt{x}$

sol $\frac{dy}{dt} = 5t^4$

$$\frac{dt}{dx} = \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{dy}{dt} * \frac{dt}{dx} = 5t^4 * \frac{1}{2\sqrt{x}}$$ و

هنا السؤال $t = \sqrt{x}$

$$\therefore \frac{dy}{dx} = 5t^4 * \frac{1}{2t} = \frac{5}{2}t^3$$

$$\Rightarrow \frac{dy}{dx} = \frac{5}{2}(\sqrt{x})^3$$

$$\therefore \frac{dy}{dx} = \frac{5}{2}x^{3/2}$$

دائماً الناتج النهائي يكون بدلالة (x).

(2) If $y = F(t)$ and $x = h(t)$ Then

* نشتق كل منهما على حدة . وللحصول على $\frac{dy}{dx}$ نجري
عملية القسمة .

$$\boxed{\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}}$$

ex (2): Find $\frac{dy}{dx}$ if $y = 1 - t^2$ and $x = 4t^2$

<u>sol</u> $\frac{dy}{dt} = -2t$

$$\frac{dx}{dt} = 8t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2t}{8t} = -\frac{1}{4}$$

ex (3)

find $\frac{dy}{dx}$ if $y = \frac{t^2}{t^2+1}$ and $t = \sqrt{2x+1}$

Sol

$$\frac{dy}{dt} = \frac{(t^2+1) * 2t - t^2 * 2t}{(t^2+1)^2}$$

$$\frac{dy}{dt} = \frac{2t^3+2t-2t^3}{(t^2+1)^2} = \frac{2t}{(t^2+1)^2}$$

$$\frac{dt}{dx} = \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} * \frac{dt}{dx} = \frac{2t}{(t^2+1)^2} * \frac{1}{\sqrt{2x+1}}$$

لان نحول السؤال بدلالة (x)

$$t = \sqrt{2x+1} \longrightarrow t^2 = 2x+1$$

$$\frac{dy}{dx} = \frac{2\sqrt{2x+1}}{(2x+1+1)^2} * \frac{1}{\cancel{\sqrt{2x+1}}} = \frac{2}{(2x+2)^2}$$

نخرج 2 من العوامل

$$= \frac{\cancel{2}}{\cancel{4}(x+1)^2} = \frac{1}{2(x+1)^2}$$



REDMI NOTE 8 PRO
64MP QUAD CAMERA

ex (24) find $\frac{dy}{dx}$. if $y = \left(\frac{t-1}{t+1}\right)^2$ and

$x = \frac{1}{t^2} - 1$ at $(t=2)$

Sol

$$\frac{dy}{dt} = 2\left(\frac{t-1}{t+1}\right) * \left(\frac{(t+1)*1 - (t-1)*1}{(t+1)^2}\right)$$

$$= 2\left(\frac{t-1}{t+1}\right) * \left(\frac{t+1 - t+1}{(t+1)^2}\right)$$

$$= 2\left(\frac{t-1}{t+1}\right) * \left(\frac{2}{(t+1)^2}\right)$$

نعوض عن قيمة $t=2$

$$\frac{dy}{dt} = \frac{4(t-1)}{(t+1)^3} = \frac{4(2-1)}{(2+1)^3} = \boxed{\frac{4}{27}}$$

$$\frac{dx}{dt} = \frac{t^2*0 - 1*2t}{t^4} - 0 = \frac{-2t}{t^4} = \frac{-2}{t^3}$$

$$\therefore \frac{dx}{dt} = \frac{-2}{(2)^3} = \frac{-2}{8} = -\frac{1}{4}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{4}{27}}{\frac{-1}{4}} = \frac{4}{27} * -4$$

$$\therefore \frac{dy}{dx} = -\frac{16}{27}$$ ans

ملاحظة ٣ لايجاد المشتقة الثانية $\left(\frac{d^2y}{dx^2}\right)$ للقاعدة الثانية وهي عندما يعطى $[\, y=f(t) \,,\, x=f(t) \,]$ من تطبيق القانون التالي بعد استخراج $\frac{dy}{dx}$

اذن :

$$\boxed{\frac{d^2y}{dx^2} = \frac{\text{مشتقة المشتقة لـ } \left(\frac{dy}{dx}\right)}{\frac{dx}{dt} \text{ كما هي}}}$$

**ex 5** find $\frac{d^2y}{dx^2}$ if $y = t-1 \,,\, x = 2t^2$

sol

$$\frac{dy}{dt} = 1 \,,\quad \frac{dx}{dt} = 4t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{4t}$$

الآن نجد مشتقة المشتقة لـ $\frac{dy}{dx}$

$$\frac{d^2\left(\frac{dy}{dx}\right)}{dt} = \frac{4t*0 - 1*4}{(4t)^2} = \frac{-4}{16t^2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}} = \frac{-\frac{\cancel{4}}{16t^2}}{\cancel{4}t} =$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-1}{16t^3}$$

نحول السؤال بدلالة x

$$\text{when } x = 2t^2 \,,\quad t = \frac{\sqrt{x}}{\sqrt{2}} \,,\quad t^3 = \frac{(x^{1/2})^3}{((2)^{1/2})^3}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{16 * \frac{x^{3/2}}{2^{3/2}}} = \frac{-\sqrt{(2)^3}}{16\sqrt{x^3}}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-2\sqrt{2}}{16\sqrt{x^3}} = \frac{-\sqrt{2}}{8\sqrt{x^3}} \quad 2^{3/2}$$

ex 6 find $\frac{dy}{dx}$ if $y=t^2$ and $t=\sqrt{h}$ and $h=x^2$

Sol

$\frac{dy}{dt}=2t$ , $\frac{dt}{dh}=\frac{1}{2\sqrt{h}}$ , $\frac{dh}{dx}=2x$

$\frac{dy}{dx}=\frac{dy}{dt}*\frac{dt}{dh}*\frac{dh}{dx}=2t*\frac{1}{2\sqrt{h}}*2x$

when $t=\sqrt{h}$ , $h=x^2 \Rightarrow t=x$

when $h=x^2 \Rightarrow \sqrt{h}=x$

$\therefore \frac{dy}{dx}=2x*\frac{1}{2x}*2x=2x$

ex 7 find $\frac{d^2y}{dx^2}$ if $y=3\cos\theta+\cos^3\theta$ and $x=\sin^3\theta$

Sol

$\frac{dy}{d\theta}=3*-\sin\theta+3\cos^2\theta.(-\sin\theta)$

$\frac{dy}{d\theta}=-3\sin\theta-3\cos^2\theta\sin\theta$

$\frac{dx}{d\theta}=3\sin^2\theta.\cos\theta$

$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-3\sin\theta-3\cos^2\theta.\sin\theta}{3.\sin^2\theta.\cos\theta}$

$\cancel{\frac{-1}{3\sin^2\theta}\cos^2\theta.\cos\theta}$

$=\frac{1}{3}\cancel{\cos^2\ldots}$

$$= \frac{-3\sin\theta(1+\cos^2\theta)}{3\sin^2\theta\cdot\cos\theta} = \frac{-(-\sin^2\theta)}{\sin\theta\cdot\cos\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta\cdot\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

الآن نجد $\frac{d^2y}{dx^2}$

$$\frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \sec^2\theta \Rightarrow \cancel{\frac{d^2y}{dx^2}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{d\theta}} = \frac{\sec^2\theta}{3\sin^2\theta\cdot\cos\theta}$$

$$= \frac{1}{3\sin^2\theta\cdot\cos^2\theta\cdot\cos\theta} = \frac{1}{3}\csc^2\theta\cdot\sec^3\theta$$

بعدها نحول السؤال بدلالة $x$

تكملة (واجب)

## Equations of straight Line

معادلة المماس

ملحوظة: لايجاد معادلة المماس لمنحنى الدالة نطبق القانون

$$y - y_0 = m(x - x_0)$$

* لايجاد معادلة المماس يجب توفر ميل ونقطة.
* الميل هو يمثل المشتقة الاولى للدالة.

ex ① Find the equation of the Tangent line to the Curve $y = \cos\sqrt{x}$ at $(\pi^2, 3)$

Sol

$$\frac{dy}{dx} = -\sin\sqrt{x} * \frac{1}{2\sqrt{x}} = -\frac{1}{2} * \frac{\sin\sqrt{x}}{\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = m = -\frac{1}{2} * \frac{\sin\sqrt{\pi^2}}{\sqrt{\pi^2}}$$

$$= -\frac{1}{2} * \frac{\sin\pi}{\pi} = -\frac{1}{2} * \frac{0}{\pi} = 0$$

$$\therefore y - y_0 = m(x - x_0)$$

$$y - 3 = 0(x - \pi^2)$$

$\therefore y - 3 = 0$ The equation of the Tangent

* عندما يطلب معادلة المماس لمنحنى الدالة وقال أنها تمر بالنقطتين $P_1(x_1, y_1)$ ، $P_2(x_2, y_1)$ فإننا نستخرج الميل كالتالي:

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

وإحدى النقطتين تحقق معادلة المماس

ex ② Find the equation for the line that passes through points $P_1(-2, 0)$ and $P_2(2, -2)$

**sol**

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{2 - (-2)} = \frac{-2}{4} = -\frac{1}{2}$$

$$y - y_0 = m(x - x_0)$$

أول نقطة تحقق معادلة المماس (-2,0)

$$\therefore y - 0 = -\frac{1}{2}(x - (-2)) \Rightarrow y = -\frac{1}{2}x - 1$$

نضرب $2x$

$$2y = -x - 2 \Rightarrow \boxed{2y + x + 2 = 0}$$

equation of tangent

ex ③: find the equation of the tangent to the curve $y=(1+\sqrt[3]{x})^3$ at $\boxed{x=1}$

Sol $y=(1+\sqrt[3]{1})^3=(1+1)^3=(2)^3=8$

The point : $(1, 8)$

$$\frac{dy}{dx} = 3(1+\sqrt[3]{x})^2 * \frac{1}{3} x^{-\frac{2}{3}} = \frac{(1+\sqrt[3]{x})^2}{x^{\frac{2}{3}}}$$

$$m = \frac{(1+\sqrt[3]{1})}{(1)^{\frac{2}{3}}} = \frac{(2)^2}{1} = 4$$

$\therefore y-y_0 = m(x-x_0) \Rightarrow y-8=4(x-1)$

$\therefore \boxed{y-4x-4=0}$ is equation of the tangent

25

* ملاحظة - المستقيمات المتوازيات لهما نفس الميل

المستقيمان المتعامدان فإن حاصل ضرب ميليهما = -1

أي أن ميل الثاني = المقلوب بسالب ميل المستقيم الأول وبالعكس

**ex ④** Find The equation for the line through

① $p(2,1)$ parallel to $L: y = x^2 + 2$

② an equation for the line through $p$ perpendicular to $L$.

عمودي

**Sol**

① $\frac{dy}{dx} = 2x \Rightarrow m_1 = 2x$ (لأنهما متوازيان)

$\therefore m_1 = 2 * 2 = 4$

$\Rightarrow y - y_0 = m_1(x - x_0) \Rightarrow y - 1 = 4(x - 2)$

$y - 1 = 4x - 8$

$\therefore y - 4x - 1 + 8 = 0 \Rightarrow \boxed{y - 4x + 7 = 0}$

② $m_2 = -\frac{1}{4}$ [لأنهما متعامدان]

$\therefore y - y_0 = m_2(x - x_0)$

$y - 1 = -\frac{1}{4}(x - 2) \Rightarrow y - 1 = -\frac{1}{4}x + \frac{1}{2}$ * [ضرب $4 \times$] للطرفين

$4(y - 1) = -1x + 2 \Rightarrow 4y - 4 = -x + 2$

$4y + x - 4 - 2 = 0 \Rightarrow \boxed{4y + x - 6 = 0}$

* ملاحظة: عندما يعطي أو يطلب الزاوية ($\phi$) في هذا الموضوع
فإن ($m = \tan\phi$)

ex ⑤ Find the equation of the tangent through the point $p(1,4)$ with the angle of inclination $\phi = 60$

Sol

$m = \tan\phi \Rightarrow m = \tan 60 = \sqrt{3}$

$\therefore y - y_0 = m(x - x_0) \Rightarrow y - 4 = \sqrt{3}(x-1)$

$y - 4 = \sqrt{3}x - \sqrt{3} \Rightarrow y - 4 - \sqrt{3}x + \sqrt{3} = 0$

$\therefore y - \sqrt{3}x - 4 + \sqrt{3} = 0$ equation

ex ⑥ Find the equation of the straight line through $A(7,5)$ perpendicular (عمودي) to the line AB whose the equation is $[3x + 4y - 16 = 0]$

Sol

$3 + 4 * \frac{dy}{dx} - 0 = 0 \Rightarrow 4\frac{dy}{dx} = -3 \Rightarrow \frac{dy}{dx} = \frac{-3}{4}$

$\therefore m = \frac{4}{3}$ ميل العمودي على المستقيم

$\therefore y - y_0 = m(x - x_0) \Rightarrow y - 5 = \frac{4}{3}(x - 7)$

(نضرب × 3)

$3(y-5) = 4(x-7) \Rightarrow 3y - 15 = 4x - 28$

$3y - 4x - 15 + 28 = 0 \Rightarrow 3y - 4x + 13 = 0$ eq.

ex. ⑦

Find the equation of the Line tangent to the curve $x = 2 + \sec\phi$ and $y = 1 + 2\tan\theta$ at $\phi = \frac{\pi}{6}$

<u>Sol</u>

$$\frac{dx}{d\theta} = \sec\theta \cdot \tan\theta .$$

$$\frac{dy}{d\theta} = 2\sec^2\theta .$$

$$\frac{dy}{dx} = \frac{2\sec^2\theta}{\sec\theta \cdot \tan\theta} = \frac{2\sec\theta}{\tan\theta} = \frac{2 * \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{2}{\sin\theta} = 2\csc\theta .$$

$$m = 2\csc\theta = 2 * \frac{1}{\sin\frac{\pi}{6}} = 2 * \frac{1}{\frac{1}{2}} = 2 * 2 = 4$$

نعوض (θ) في (y,x) لنحصل على النقطة

$$x = 2 + \frac{1}{\cos\frac{\pi}{6}} \Rightarrow x = 2 + \frac{1}{\frac{\sqrt{3}}{2}} = 2 + \frac{2}{\sqrt{3}} = 3.154$$

$$y = 1 + 2\tan\frac{\pi}{6} = 1 + 2 * \frac{1}{\sqrt{3}} = 1 + \frac{2}{\sqrt{3}} = 2.154$$

$\therefore$ The point $(3.154, 2.154)$

$$y - y_0 = m(x - x_0) \Rightarrow y - 2.154 = 4(x - 3.154)$$

$$y - 2.154 = 4x - 4 * 3.154$$

$$y - 4x + 4 * 3.154 - 2.154 = 0$$

$$\Rightarrow \boxed{y - 4x + 10.362 = 0} \text{ eq.}$$

28

REDMI NOTE 8 PRO
64MP QUAD CAMERA

مشتقات الدوال الأسية واللوغاريتمية

## $\boxed{1}$ Exponetial Functions ($e^x$)

Rules of ($e^x$)

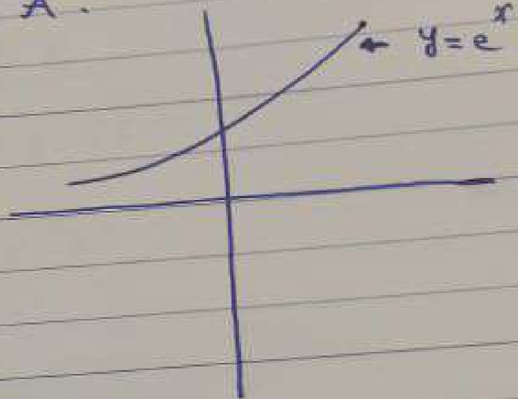
① $e^x \cdot e^y = e^{x+y}$

② $\dfrac{e^x}{e^y} = e^{x-y}$

③ $e^{-x} = \dfrac{1}{e^x}$

④ $\ln e^x = x$ , $e^{\ln x} = x$.

Graphs of $e^x$




$D_{e^x} = R$

$R_{e^x} = \{y : y > 0\}$

قاعدة اشتقاق (Exponential function)

$$\boxed{\frac{d}{dx}\left(e^{F(x)}\right) = e^{F(x)} * F'(x)}$$

اي استقاقها نزلها نفسها * مشتقة الأس.

ex ① Find $\frac{dy}{dx}$ . for $y = e^{3x}$

Sol $\frac{dy}{dx} = e^{3x} * 3 = 3e^{3x}$

ex ② Find $\frac{dy}{dx}$ . for $y = e^{\sin x}$

Sol $\frac{dy}{dx} = e^{\sin x} * \cos x$

$= \cos x \cdot e^{\sin x}$

ex 3 find $\frac{dy}{dx}$ . for $y = e^{\sqrt{1+5x^2}}$

Sol

$$\frac{dy}{dx} = e^{\sqrt{1+5x^2}} * \frac{\cancel{10}\, x}{\cancel{2}\sqrt{1+5x^2}}$$

$$= \frac{5x}{\sqrt{1+5x^2}} \cdot e^{\sqrt{1+5x^2}}$$

ex 4 Find $\frac{dy}{dx}$ . for $y = \sec^2(e^{4x})$

Sol $\frac{dy}{dx} = 2\sec(e^{4x}) \cdot \sec(e^{4x}) \cdot \tan(e^{4x})$

$* e^{4x} * 4$

ex 5) Find $\frac{dy}{dx}$ for $y = e^{x^2 * \tan x}$

Sol

$$\frac{dy}{dx} = e^{x^2 * \tan} * [x^2 * \sec^2 x + \tan x * 2x]$$

2 The Function $(a^x)$

* Rules of $(a^x)$ for any number of $x$ and $y$.

① $a^x * a^y = a^{x+y}$ if $a > 0$

② $a^{-x} = \frac{1}{a^x}$

③ $\frac{a^x}{a^y} = a^{x-y}$

④ $(a^x)^y = a^{xy}$

قاعدة اشتقاق [دالة $a^x$]

$$\boxed{\frac{d}{dx}\left(a^{F(x)}\right) = a^{F(x)} * F'(x) * \ln a}$$

اي مشتقها هي نزلها نفسها * مشتقة الاس

* (ln الاساس)

ex ① Find $\frac{dy}{dx}$ for $y = 2^{5x}$

sol $\frac{dy}{dx} = 2^{5x} * 5 * \ln 2$

$= 5 * 2^{5x} * \ln 2$

$= 5 \cdot 2^{5x} \ln 2$

ex 2) $y = 2^x \cdot 3^x$ find $\frac{dy}{dx}$

sol $\frac{dy}{dx}$ $2^x * [3^x * 1 * \ln 3] + 3^x * [2^x * 1 * \ln 2]$

ex. 3) $y = 7^{\cos\sqrt{2x+3}}$

sol $\frac{dy}{dx} = 7^{\cos\sqrt{2x+3}} * -\sin\sqrt{2x+3} * \frac{2}{2\sqrt{2x+3}} * \ln 7$

$\therefore \frac{dy}{dx} = \frac{-7^{\cos\sqrt{2x+3}}}{\sqrt{2x+3}} \cdot \sin\sqrt{2x+3} \cdot \ln 7$

ex 4) $y = 5^{\csc x^3}$

sol $\frac{dy}{dx} = 5^{\csc x^3} * -\csc x^3 \cdot \cot x^3 \cdot 3x^2 \cdot * \ln 5$

ex 5) Find $\frac{dy}{dx}$ for $y = 3^{(x^2 - e^{6x^2})}$

sol $\frac{dy}{dx} = 3^{(x^2 - e^{6x^2})} * (2x - e^{6x^2} * 12x) * \ln 3$

## The logarithmic functions

(الدوال اللوغارتمية)

1- $\log(x \cdot y) = \log x + \log y$

2- $\log\left(\frac{x}{y}\right) = \log x - \log y$

3- $\log x^{a} = a \log x$

4- $\log_e^{F(x)} = \frac{\ln F(x)}{\ln e} = \ln F(x)$

[ لأن (e) هو معكوس (Ln) لذلك احدهما يلغي الآخر ]

5. $\log_a^{x} = \frac{\ln x}{\ln a}$ , $a \neq 1$ and x are positive

6- $\log_a^{a} = 1$ , $\log_b^{b} = 1$

* مشتقة الدالة اللوغارتمية

$$\boxed{\frac{d}{dx}\left(\log_a F(x)\right) = \frac{1}{\ln a} * \frac{1}{F(x)} * F'(x)}$$ حفظ

ex: find $\frac{dy}{dx}$ for $y = \log_4^{\sin x}$

Sol: $\frac{dy}{dx} = \frac{1}{\ln 4} * \frac{1}{\sin x} * \cos x$

$= \frac{\cos x}{\sin x} * \frac{1}{\ln 4} = \frac{\cot x}{\ln 4}$

ex ② find $\frac{dy}{dx}$ for $y = \log_7 \sqrt{x+1}$

Sol $\frac{dy}{dx} = \frac{1}{\ln 7} * \frac{1}{\sqrt{x+1}} * \frac{1}{2\sqrt{x+1}}$

$\therefore \frac{dy}{dx} = \frac{1}{2\ln 7 * (x+1)}$

ex ③ Solve for x , $\log_{10}(1+x) = 3$

Sol $\log_{10}(1+x) = 3 \Rightarrow \log_{10}(1+x) = \log_{10} 10^3$

* ملاحظة في هذه الحالة نشيل ( log ) المتشابه + نحل الطرف الايسر

$\log_{10}(1+x) = \log_{10} 10^3 \quad \div (\log_{10})$

$\Rightarrow 1+x = 10^3 \Rightarrow 1+x = 1000 \Rightarrow x = 999$

ex find the value x , $\log_5 (5^{2x}) = 8$

Sol $\log_5 (5)^{2x} = \log_5 5^8 \quad - \quad (\div \log_5)$

$5^{2x} = 5^8 \Rightarrow 2x = 8 \Rightarrow \boxed{x = 4}$

ex ⑤ Solve for $x$, $\log_{10} x^2 + \log_{10} x = 30$

*{ملاحظة} عند وجود جمع او طرح في موضوع (log) وطلب حل المعادلة فيجب اننا نرجعه الى اصله ونكمل الحل

**Sol** $\log_{10}(x^2 \cdot x) = 30 \Longrightarrow \log_{10} x^3 = 30$

$\log_{10} x^3 = \log_{10} 10^{30}$ $(\div \log 10)$

$x^3 = 10^{30} \Longrightarrow 10^{10}$

ex ⑥ Solve for $x$, $\log_{10} x^{\frac{3}{2}} - \log_{10} \sqrt{x} = 5$

**Sol** نرجعه الى الاصل

$\log_{10}\left(\frac{x^{3/2}}{x^{\frac{1}{2}}}\right) = 5$

$\Longrightarrow \log_{10} x = \log_{10} 10^5$ $\div(\log_{10})$

$x = 10^5$

The natural logarithm Function [ln دالة]

Rules of $\ln x$

1- $\ln(x \cdot y) = \ln x + \ln y$

2- $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

3- $\ln \frac{1}{x} = \ln 1 - \ln x = -\ln x$

4- $\ln x^n = n \ln x$

* لإيجاد المشتقة لدالة (ln) نطبق القاعدة التالية

$$\boxed{\frac{d}{dx}(\ln u) = \frac{1}{u} * \frac{du}{dx}}$$

ex ① Find $\frac{dy}{dx}$ for $y = \ln x^4$

Sol

$y = \ln x^4 \Rightarrow 4 \ln x$

$\frac{dy}{dx} = 4 * \frac{1}{x} * 1 = \frac{4}{x}$

ex ② find $\frac{dy}{dx}$ . for $y = \ln \cos x$

Sol $\frac{dy}{dx} = \frac{1}{\cos x} * -\sin x = \frac{-\sin x}{\cos x}$

$= -\tan x$

ex ③

$y = \ln(x^2 + \cos x)$

Sol $\frac{dy}{dx} = \frac{1}{x^2 + \cos x} * (2x - \sin x)$

ex ④: $y = \ln^3(\ln x)$

Sol $y = 3 \ln(\ln x)$

$\frac{dy}{dx} = 3 * \frac{1}{\ln x} * \frac{1}{x} * 1$

$= \frac{3}{x \ln x}$

ex ⑤ Find $\frac{dy}{dx}$ . for $y = \tan(\ln \sqrt{x})$

Sol $\frac{dy}{dx} = \sec^2(\ln \sqrt{x}) * \frac{1}{\sqrt{x}} * \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2x} \cdot \sec^2(\ln \sqrt{x})$

ex ⑥ Find $\frac{dy}{dx}$ for $y = \ln(x^2+2)^2 \cdot \cos x$

Sol $y = 2\ln(x^2+2)\cdot\cos x$

$\frac{dy}{dx} = 2\ln(x^2+2) * -\sin x + \cos x * 2 * \frac{1}{x^2+2} * 2x$

$= -2\ln(x^2+2)\cdot\sin x + \cos x * \frac{4x}{x^2+2}$

$\frac{dy}{dx} = \cos x * \frac{4x}{x^2+2} - 2\ln(x^2+2)\cdot\sin x$

ex ⑦ solve for x ; $\ln 4x - 3\ln x^3 = \ln 2$

Sol $\ln\left(\frac{4x}{(x^2)^3}\right) = \ln 2 \Rightarrow \ln\left(\frac{4x}{x^6}\right) = \ln 2$

$\ln\left(\frac{4}{x^5}\right) = \ln 2$ $(\div \ln)$

$\frac{4}{x^5} = 2 \Rightarrow x^5 = \frac{4}{2} = 2$

$\Rightarrow x^5 = 2 \Rightarrow \boxed{x = \sqrt[5]{2}}$

ex ⑧ solve for x ; $\ln\left(\frac{1}{x}\right) + \ln(2x)^3 = \ln 3$

Sol $\ln\left(\frac{1}{\cancel{x}} \cdot * 2 x^{\cancel{3}}\right) = \ln 3$

$\ln(2x^2) = \ln 3$ $(\div \ln)$

$2x^2 = 3 \Rightarrow x^2 = \frac{3}{2} \Rightarrow x = \frac{\sqrt{3}}{\sqrt{2}}$

ex (9) Solve for $x$. If $y = 2^x$ and $y = 3^{x+1}$

Sol $2^x = 3^{x+1}$ (نأخذ ln للطرفين)

$\ln 2^x = \ln 3^{(x+1)} \Rightarrow x \ln 2 = (x+1) \ln 3$

$x \ln 2 = x \ln 3 + \ln 3 \Rightarrow x \ln 2 - x \ln 3 = \ln 3$

$x(\ln 2 - \ln 3) = \ln 3 \Rightarrow x(\ln \frac{2}{3}) = \ln 3$

$\therefore x = \dfrac{\ln 3}{\ln \frac{2}{3}}$

ex (10) solve for $x$ ; $e^{2x} + 2e^{x} + 1 = 9$

Sol $e^{2x} + e^{2x} = 9 - 1 \Rightarrow e^{2x} + e^{2x} = 8$

$2e^{2x} = 8 \Rightarrow e^{2x} = 4 \Rightarrow e^{x} = 2$ (نأخذ ln للطرفين)

$\ln e^{x} = \ln 2 \Rightarrow x = \ln 2$

ex (11) find $\frac{dy}{dx}$ for $y = x^x$

Sol $y = x^x$ ( نأخذ ln للطرفين )

$\ln y = \ln x^x \Rightarrow \ln y = x \ln x$

$\frac{1}{y} \cdot \frac{dy}{dx} = x * \frac{1}{x} + \ln x * 1$

$\Rightarrow \frac{1}{y} * \frac{dy}{dx} = 1 + \ln x$

$\frac{dy}{dx} = \frac{1+\ln x}{\frac{1}{y}} = y\,[1+\ln x]$

$\therefore \frac{dy}{dx} = x^x\,[1+\ln x]$

ex (12) solve for $x$: $\frac{e^{x^2} \cdot (e^x)^3}{e^2} = 1$

Sol $\frac{e^{x^2} \cdot (e^x)^3}{e^2} = 1$

$\frac{e^{2x} \cdot e^{3x}}{e^2} = 1 \Rightarrow e^{2x} \cdot e^{3x} = e^2$

$e^{2x+3x} = e^2$ نأخذ (ln) للطرفين

$\cancel{\ln}\ \ln e^{2x+3x} = \ln e^2 \Rightarrow 2x+3x = 2$

$\therefore 5x = 2 \Rightarrow \boxed{x = \frac{2}{5}}$